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APPLIED TO QUESTIONS OF

INTEREST, ANNUITIES, LIFE ASSURANCE

AND

GENERAL COMMERCE

WITH

VARIOUS TABLES

BY WHICH ALL CALCULATIONS MAY BE GREATLY FACILITATED.

BY W. HIPSLEY.

THIRD EDITION.

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PREFACE.

ALTHOUGH many Treatises on Arithmetic are already before the public, they are, for the most part, open to this objection, that they seek rather to invent labor for the student, by multiplying rules and examples, than to economise it by the most concise methods. The present work, which is not intended exclusively for a school book, nor yet to supersede the more useful of the elementary works already published, will, it is confidently believed, afford advantages to the accountant, merchant, and private student, which are not to be found in any other.

It is said, "that railways have been used in this country for more than two centuries:" the recent and rapid extension of the system is a proof that the practical application and improvement of well-known arts is often of more value than an absolutely new invention. In like manner, the simplest form of fractional arithmetic, that of decimals, though so ancient, and to the present day so universal a branch of school-boy lore, has so unaccountably fallen into disuse as to be all but forgotten in after years.

Another powerful motive for increasing in the public mind an interest in this mode of calculation is, that we thereby diffuse a knowledge of the great commercial benefits to be derived from an universal decimal scale of money, weights, and measures. Thus, we may ultimately obtain some parliamentary enactment which may rescue this country from the disgrace of being behind some other nations in the practical application of general principles.

By giving only one or two examples, as a formula for each kind of calculation, we are enabled so to extend the work as to include Compound Interest, Annuities, Life Assurance, and almost every computation that comes within the sphere of ordinary arithmetic; thus comprising in one small volume, of which the tables alone are worth the cost, what has hitherto been diffused through many large and elaborate works, at a proportionally large expense.

Every calculation in this work is reduced or reducible to one form—that of a simple equation; it has been the aim of the writer so to elucidate this method, that the accountant shall find no difficulty in applying the principle to any question whatever, though its formula be not found in these pages. By means of the decimal tables alone, which have never before been published at a moderate price, a vast amount of labor may be saved. All fractions of English money and time are already reduced to decimals in Tables I and II., to which is added a new table of the decimals of the hundredweight, and the usual tabular values of compound interest and annuities at various rates per cent. The facilities of the decimal system may thus, in some degree, be anticipated.

These advantages, together with that of reducing the innumerable rules of common arithmetic to the simple form of an equation (without assuming the merit of a new discovery), form, at least, an unique combination, affording all the facilities that could reasonably be expected, even from a *new calculus,* applicable to the ordinary purposes of commercial and scientific investigation.

The great extension of prudential assurance against death and the various contingencies of life, which has recently taken place, renders a knowledge of this philanthropic subject almost a necessary part of a liberal education. It is, moreover, desirable to prepare the public mind for the discussion of the great question of financial reform, forced upon

us by the necessities of the times; this can be accomplished only by popular treatises on the higher branches of calculation, of which it is hoped the present work will prove neither the *last* nor the *best*.

It is disgraceful to our boasted civilization, that pauperism and destitution should be the lot of any but the incorrigibly vicious and criminal; for it is well known to those accustomed to the calculation of annuities, that the poor rates might be entirely extinguished by an extension of its principles to the labouring classes*.

The profitable employment of the masses is now an European question. Though, on the Continent, the attempts to realize it may assume the distorted forms of "socialism" or communism, in themselves impracticable, it requires little sagacity to predict that the true solution of the problem, the universal scheme of co-operative labor and economy of capital, must ultimately be founded on principles developed from those of life assurance and annuities.

The unprecedented success of the Industrial Exhibition of 1851 is a striking illustration of the wonderful effects of coöperative labor, or capital, which is the result of labor, when *judiciously* applied to a useful end.

The art of arithmetic and the art of war, are, unfortunately, very closely connected. When money has been demanded for the purpose of destroying our fellow men, under the plea of avenging national honor, restoring the balance of power, natural enemies, or any vulgar delusion of the day, there has hitherto been little difficulty in raising loans to any required amount, to be repaid in *any way*, or in *no way*, as posterity might think proper There can be no greater proof than this of financial ignorance, equalled only by moral imbecility; for a mere tithe of those enormous sums, which have been utterly wasted by an art which belongs to barbarous ages,

* See "Arithmetic of Annuities," by Edward Baylis.

would have been sufficient, if applied in productive industry, to extinguish all the pauperism in Europe.

"——— dehinc absistere bello,
Oppida cœperunt munire, et ponere leges."—HOR. *Sat.*

The writer, having had some experience in the business of a Life Assurance Office, has been careful to select the most useful formulæ for all calculations that usually occur in this department. These are adapted to the Northampton Table of Mortality, a copy of which is included in the work, but are of course equally applicable to any other.

W. H.

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PART II.

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TABLE I.

Decimals corresponding with every Farthing in the Pound.

s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.
0	¼	·00104167	1	0¼	·05104167	2	0¼	·10104167	3	0¼	·15104167
0	½	·00208333	1	0½	·05208333	2	0½	·10208333	3	0½	·15208333
0	¾	·003125	1	0¾	·053125	2	0¾	·103125	3	0¾	·153125
0	1	·00416667	1	1	·05416667	2	1	·10416667	3	1	·15416667
0	1¼	·00520833	1	1¼	·05520833	2	1¼	·10520833	3	1¼	·15520833
0	1½	·00625	1	1½	·05625	2	1½	·10625	3	1½	·15625
0	1¾	·00729167	1	1¾	·05729167	2	1¾	·10729167	3	1¾	·15729167
0	2	·00833333	1	2	·05833333	2	2	·10833333	3	2	·15833333
0	2¼	·009375	1	2¼	·059375	2	2¼	·109375	3	2¼	·159375
0	2½	·01041667	1	2½	·06041667	2	2½	·11041667	3	2½	·16041667
0	2¾	·01145833	1	2¾	·06145833	2	2¾	·11145833	3	2¾	·16145833
0	3	·0125	1	3	·0625	2	3	·1125	3	3	·1625
0	3¼	·01354167	1	3¼	·06354167	2	3¼	·11354167	3	3¼	·16354167
0	3½	·01458333	1	3½	·06458333	2	3½	·11458333	3	3½	·16458333
0	3¾	·015625	1	3¾	·065625	2	3¾	·115625	3	3¾	·165625
0	4	·01666667	1	4	·06666667	2	4	·11666667	3	4	·16666667
0	4¼	·01770833	1	4¼	·06770833	2	4¼	·11770833	3	4¼	·16770833
0	4½	·01875	1	4½	·06875	2	4½	·11875	3	4½	·16875
0	4¾	·01979167	1	4¾	·06979167	2	4¾	·11979167	3	4¾	·16979167
0	5	·02083333	1	5	·07083333	2	5	·12083333	3	5	·17083333
0	5¼	·021875	1	5¼	·071875	2	5¼	·121875	3	5¼	·171875
0	5½	·02291667	1	5½	·07291667	2	5½	·12291667	3	5½	·17291667
0	5¾	·02395833	1	5¾	·07395833	2	5¾	·12395833	3	5¾	·17395833
0	6	·025	1	6	·075	2	6	·125	3	6	·175
0	6¼	·02604167	1	6¼	·07604167	2	6¼	·12604167	3	6¼	·17604167
0	6½	·02708333	1	6½	·07708333	2	6½	·12708333	3	6½	·17708333
0	6¾	·028125	1	6¾	·078125	2	6¾	·128125	3	6¾	·178125
0	7	·02916667	1	7	·07916667	2	7	·12916667	3	7	·17916667
0	7¼	·03020833	1	7¼	·08020833	2	7¼	·13020833	3	7¼	·18020833
0	7½	·03125	1	7½	·08125	2	7½	·13125	3	7½	·18125
0	7¾	·03229167	1	7¾	·08229167	2	7¾	·13229167	3	7¾	·18229167
0	8	·03333333	1	8	·08333333	2	8	·13333333	3	8	·18333333
0	8¼	·034375	1	8¼	·084375	2	8¼	·134375	3	8¼	·184375
0	8½	·03541667	1	8½	·08541667	2	8½	·13541667	3	8½	·18541667
0	8¾	·03645833	1	8¾	·08645833	2	8¾	·13645833	3	8¾	·18645833
0	9	·0375	1	9	·0875	2	9	·1375	3	9	·1875
0	9¼	·03854167	1	9¼	·08854167	2	9¼	·13854167	3	9¼	·18854167
0	9½	·03958333	1	9½	·08958333	2	9½	·13958333	3	9½	·18958333
0	9¾	·040625	1	9¾	·090625	2	9¾	·140625	3	9¾	·190625
0	10	·04166667	1	10	·09166667	2	10	·14166667	3	10	·19166667
0	10¼	·04270833	1	10¼	·09270833	2	10¼	·14270833	3	10¼	·19270833
0	10½	·04375	1	10½	·09375	2	10½	·14375	3	10½	·19375
0	10¾	·04479167	1	10¾	·09479167	2	10¾	·14479167	3	10¾	·19479167
0	11	·04583333	1	11	·09583333	2	11	·14583333	3	11	·19583333
0	11¼	·046875	1	11¼	·096875	2	11¼	·146875	3	11¼	·196875
0	11½	·04791667	1	11½	·09791667	2	11½	·14791667	3	11½	·19791667
0	11¾	·04895833	1	11¾	·09895833	2	11¾	·14895833	3	11¾	·19895833
1	0	·05	2	0	·1	3	0	·15	4	0	·2

TABLE I.—*continued.*

Decimals corresponding with every Farthing in the Pound.

s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.
4	0¼	·20104167	5	0¼	·25104167	6	0¼	·30104167	7	0¼	·35104167
4	0½	·20208333	5	0½	·25208333	6	0½	·30208333	7	0½	·35208333
4	0¾	·203125	5	0¾	·253125	6	0¾	·303125	7	0¾	·353125
4	1	·20416667	5	1	·25416667	6	1	·30416667	7	1	·35416667
4	1¼	·20520833	5	1¼	·25520833	6	1¼	·30520833	7	1¼	·35520833
4	1½	·20625	5	1½	·25625	6	1½	·30625	7	1½	·35625
4	1¾	·20729167	5	1¾	·25729167	6	1¾	·30729167	7	1¾	·35729167
4	2	·20833333	5	2	·25833333	6	2	·30833333	7	2	·35833333
4	2¼	·209375	5	2¼	·259375	6	2¼	·309375	7	2¼	·359375
4	2½	·21041667	5	2½	·26041667	6	2½	·31041667	7	2½	·36041667
4	2¾	·21145833	5	2¾	26145833	6	2¾	·31145833	7	2¾	·36145833
4	3	·2125	5	3	·2625	6	3	·3125	7	3	·3625
4	3¼	·21354167	5	3¼	·26354167	6	3¼	·31354167	7	3¼	·36354167
4	3½	·21458333	5	3½	·26458333	6	3½	·31458333	7	3½	·36458333
4	3¾	·215625	5	3¾	·265625	6	3¾	·315625	7	3¾	·365625
4	4	·21666667	5	4	·26666667	6	4	·31666667	7	4	·36666667
4	4¼	·21770833	5	4¼	·26770833	6	4¼	·31770833	7	4¼	·36770833
4	4½	·21875	5	4½	·26875	6	4½	·31875	7	4½	·36875
4	4¾	·21979167	5	4¾	·26979167	6	4¾	·31979167	7	4¾	·36979167
4	5	·22083333	5	5	·27083333	6	5	·32083333	7	5	·37083333
4	5¼	·221875	5	5¼	·271875	6	5¼	·321875	7	5¼	·371875
4	5½	·22291667	5	5½	·27291667	6	5½	·32291667	7	5½	·37291667
4	5¾	·22395833	5	5¾	·27395833	6	5¾	·32395833	7	5¾	·37395833
4	6	·225	5	6	·275	6	6	·325	7	6	·375
4	6¼	·22604167	5	6¼	·27604167	6	6¼	·32604167	7	6¼	·37604167
4	6½	·22708333	5	6½	·27708333	6	6½	·32708333	7	6½	·37708333
4	6¾	·2228125	5	6¾	·278125	6	6¾	·328125	7	6¾	·378125
4	7	·22916667	5	7	·27916667	6	7	·32916667	7	7	·37916667
4	7¼	·23020833	5	7¼	·28020833	6	7¼	·33020833	7	7¼	·38020833
4	7½	·23125	5	7½	·28125	6	7½	·33125	7	7½	·38125
4	7¾	·23229167	5	7¾	·28229167	6	7¾	·33229167	7	7¾	·38229167
4	8	·23333333	5	8	·28333333	6	8	·33333333	7	8	·38333333
4	8¼	·234375	5	8¼	·284375	6	8¼	(334375	7	8¼	·384375
4	8½	·23541667	5	8½	·28541667	6	8½	·33541667	7	8½	·38541667
4	8¾	·23645833	5	8¾	·28645833	6	8¾	·33645833	7	8¾	·38645833
4	9	·2375	5	9	·2875	6	9	·3375	7	9	·3875
4	9¼	·23854167	5	9¼	·28854167	6	9¼	·33854167	7	9¼	·38854167
4	9½	·23958333	5	9½	·28958333	6	9½	·33958333	7	9½	·38958333
4	9¾	·240625	5	9¾	·290625	6	9¾	·340625	7	9¾	·390625
4	10	·24166667	5	10	·29166667	6	10	·34166667	7	10	·39166667
4	10¼	·24270833	5	10¼	·29270833	6	10¼	·34270833	7	10¼	·39270833
4	10½	·24375	5	10½	·29375	6	10½	·34375	7	10½	·39375
4	10¾	·24479167	5	10¾	·29479167	6	10¾	·34479167	7	10¾	·39479167
4	11	·24583333	5	11	·29583333	6	11	·34583333	7	11	·39583333
4	11¼	·246875	5	11¼	·296875	6	11¼	·346875	7	11¼	·396875
4	11½	·24791667	5	11½	·29791667	6	11½	·34791667	7	11½	·39791667
4	11¾	·24895833	5	11¾	·29895833	6	11¾	·34895833	7	11¾	·39895833
5	0	·25	6	0	·3	7	0	·35	8	0	·4

Decimals corresponding with every Farthing in the Pound.

s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.
8	0¼	·40104167	9	0¼	·45104167	10	0¼	·50104167	11	0¼	·55104167
8	0½	·40208333	9	0½	·45208333	10	0½	·50208333	11	0½	·55208333
8	0¾	·403125	9	0¾	·453125	10	0¾	·503125	11	0¾	·553125
8	1	·40416667	9	1	·45416667	10	1	·50416667	11	1	·55416667
8	1¼	·40520833	9	1¼	·45520833	10	1¼	·50520833	11	1¼	·55520833
8	1½	·40625	9	1½	·45625	10	1½	·50625	11	1½	·55625
8	1¾	·40729167	9	1¾	·45729167	10	1¾	·50729167	11	1¾	·55729167
8	2	·40833333	9	2	·45833333	10	2	·50833333	11	2	·55833333
8	2¼	·409375	9	2¼	·459375	10	2¼	·509375	11	2¼	·559375
8	2½	·41041667	9	2½	·46041667	10	2½	·51041667	11	2½	·56041667
8	2¾	·41145833	9	2¾	·46145833	10	2¾	·51145833	11	2¾	·56145833
8	3	·4125	9	3	·4625	10	3	·5125	11	3	·5625
8	3¼	·41354167	9	3¼	·46354167	10	3¼	·51354167	11	3¼	·56354167
8	3½	·41458333	9	3½	·46458333	10	3½	·51458333	11	3½	·56458333
8	3¾	·415625	9	3¾	·465625	10	3¾	·515625	11	3¾	·565625
8	4	·41666667	9	4	·46666667	10	4	·51666667	11	4	·56666667
8	4¼	·41770833	9	4¼	·46770833	10	4¼	·51770833	11	4¼	·56770833
8	4½	·41875	9	4½	·46875	10	4½	·51875	11	4½	·56875
8	4¾	·41979167	9	4¾	·46979167	10	4¾	·51979167	11	4¾	·56979167
8	5	·42083333	9	5	·47083333	10	5	·52083333	11	5	·57083333
8	5¼	·421875	9	5¼	·471875	10	5¼	·521875	11	5¼	·571875
8	5½	·42291667	9	5½	·47291667	10	5½	·52291667	11	5½	·57291667
8	5¾	·42395833	9	5¾	·47395833	10	5¾	·52395833	11	5¾	·57395833
8	6	·425	9	6	·475	10	6	·525	11	6	·575
8	6¼	·42604167	9	6¼	·47604167	10	6¼	·52604167	11	6¼	·57604167
8	6½	·42708333	9	6½	·47708333	10	6½	·52708333	11	6½	·57708333
8	6¾	·428125	9	6¾	·478125	10	6¾	·528125	11	6¾	·578125
8	7	·42916667	9	7	·47916667	10	7	·52916667	11	7	·57916667
8	7¼	·43020833	9	7¼	·48020833	10	7¼	·53020833	11	7¼	·58020833
8	7½	·43125	9	7½	·48125	10	7½	·53125	11	7½	·58125
8	7¾	·43229167	9	7¾	·48229167	10	7¾	·53229167	11	7¾	·58229167
8	8	·43333333	9	8	·48333333	10	8	·53333333	11	8	·58333333
8	8¼	·434375	9	8¼	·484375	10	8¼	·534375	11	8¼	·584375
8	8½	·43541667	9	8½	·48541667	10	8½	·53541667	11	8½	·58541667
8	8¾	·43645833	9	8¾	·48645833	10	8¾	·53645833	11	8¾	·58645833
8	9	·4375	9	9	·4875	10	9	·5375	11	9	·5875
8	9¼	·43854167	9	9¼	·48854167	10	9¼	·53854167	11	9¼	·58854167
8	9½	·43958333	9	9½	·48958333	10	9½	·53958333	11	9½	·58958333
8	9¾	·440625	9	9¾	·490625	10	9¾	·540625	11	9¾	·590625
8	10	·44166667	9	10	·49166667	10	10	·54166667	11	10	·59166667
8	10¼	·44270833	9	10¼	·49270833	10	10¼	·54270833	11	10¼	·59270833
8	10½	·44375	9	10½	·49375	10	10½	·54375	11	10½	·59375
8	10¾	·44479167	9	10¾	·49479167	10	10¾	·54479167	11	10¾	·59479167
8	11	·44583333	9	11	·49583333	10	11	·54583333	11	11	·59583333
8	11¼	·446875	9	11¼	·496875	10	11¼	·546875	11	11¼	·596875
8	11½	·44791667	9	11½	·49791667	10	11½	·54791667	11	11½	·59791667
8	11¾	·44895833	9	11¾	·49895833	10	11¾	·54895833	11	11¾	·59895833
9	0	·45	10	0	·5	11	0	·55	12	0	·6

Decimals corresponding with every Farthing in the Pound.

s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.
12	0¼	·60104167	13	0¼	·65104167	14	0¼	·70104167	15	0¼	·75104167
12	0½	·60208333	13	0½	·65208333	14	0½	·70208333	15	0½	·75208333
12	0¾	·603125	13	0¾	·653125	14	0¾	·703125	15	0¾	·753125
12	1	·60416667	13	1	·65416667	14	1	·70416667	15	1	·75416667
12	1¼	·60520833	13	1¼	·65520833	14	1¼	·70520833	15	1¼	·75520833
12	1½	·60625	13	1½	·65625	14	1½	·70625	15	1½	·75625
12	1¾	·60729167	13	1¾	·65729167	14	1¾	·70729167	15	1¾	·75729167
12	2	·60833333	13	2	·65833333	14	2	·70833333	15	2	·75833333
12	2¼	·609375	13	2¼	·659375	14	2¼	·709375	15	2¼	·759375
12	2½	·61041667	13	2½	·66041667	14	2½	·71041667	15	2½	·76041667
12	2¾	·61145833	13	2¾	·66145833	14	2¾	·71145833	15	2¾	·76145833
12	3	·6125	13	3	·6625	14	3	·7125	15	3	·7625
12	3¼	·61354167	13	3¼	·66354167	14	3¼	·71354167	15	3¼	·76354167
12	3½	·61458333	13	3½	·66458333	14	3½	·71458333	15	3½	·76458333
12	3¾	·615625	13	3¾	·665625	14	3¾	·715625	15	3¾	·765625
12	4	·61666667	13	4	·66666667	14	4	·71666667	15	4	76666667
12	4¼	·61770833	13	4¼	·66770833	14	4¼	·71770833	15	4¼	·76770833
12	4½	·61875	13	4½	·66875	14	4½	·71875	15	4½	·76875
12	4¾	·61979167	13	4¾	·66979167	14	4¾	·71979167	15	4¾	·76979167
12	5	·62083333	13	5	·67083333	14	5	·72083333	15	5	·77083333
12	5¼	·621875	13	5¼	·671875	14	5¼	·721875	15	5¼	·771875
12	5½	·62291667	13	5½	·67291667	14	5½	·72291667	15	5½	·77291667
12	5¾	·62395833	13	5¾	·67395833	14	5¾	·72395833	15	5¾	·77395833
12	6	·625	13	6	·675	14	6	·725	15	6	·775
12	6¼	·62604167	13	6¼	·67604167	14	6¼	·72604167	15	6¼	·77604167
12	6½	·62708333	13	6½	·67708333	14	6½	·72708333	15	6½	·77708333
12	6¾	·628125	13	6¾	·678125	14	6¾	·728125	15	6¾	·778125
12	7	·62916667	13	7	·67916667	14	7	·72916667	15	7	·77916667
12	7¼	·63020833	13	7¼	·68020833	14	7¼	·73020833	15	7¼	·78020833
12	7½	·63125	13	7½	·68125	14	7½	·73125	15	7½	·78125
12	7¾	·63229167	13	7¾	·68229167	14	7¾	·73229167	15	7¾	·78229167
12	8	·63333333	13	8	·68333333	14	8	·73333333	15	8	·78333333
12	8¼	·634375	13	8¼	·684375	14	8¼	·734375	15	8¼	·784375
12	8½	·63541667	13	8½	·68541667	14	8½	·73541667	15	8½	·78541667
12	8¾	·63645833	13	8¾	·68645833	14	8¾	·73645833	15	8¾	·78645833
12	9	·6375	13	9	·6875	14	9	·7375	15	9	·7875
12	9¼	·63854167	13	9¼	·68854167	14	9¼	·73854167	15	9¼	·78854167
12	9½	·63958333	13	9½	·68958333	14	9½	·73958333	15	9½	·78958333
12	9¾	·640625	13	9¾	·690625	14	9¾	·740625	15	9¾	·790625
12	10	·64166667	13	10	·69166667	14	10	·74166667	15	10	·79166667
12	10¼	·64270833	13	10¼	·69270833	14	10¼	·74270833	15	10¼	·79270833
12	10½	·64375	13	10½	·69375	14	10½	·74375	15	10½	·79375
12	10¾	·64479167	13	10¾	·69479167	14	10¾	·74479167	15	10¾	·79479167
12	11	·64583333	13	11	·69583333	14	11	·74583333	15	11	·79583333
12	11¼	·646875	13	11¼	·696875	14	11¼	·746875	15	11¼	·796875
12	11½	·64791667	13	11½	·69791667	14	11½	·74791667	15	11½	·79791667
12	11¾	·64895833	13	11¾	·69895833	14	11¾	·74895833	15	11¾	·79895833
13	0	·65	14	0	·7	15	0	·75	16	0	·8

Decimals corresponding with every Farthing in the Pound.

s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.	s.	d.	Decimal.
16	0¼	·80104167	17	0¼	·85104167	18	0¼	·90104167	19	0¼	·95104167
16	0½	·80208333	17	0½	·85208333	18	0½	·90208333	19	0½	·95208333
16	0¾	·803125	17	0¾	·853125	18	0¾	·903125	19	0¾	·953125
16	1	·80416667	17	1	·85416667	18	1	·90416667	19	1	·95416667
16	1¼	·80520833	17	1¼	·85520833	18	1¼	·90520833	19	1¼	·95520833
16	1½	·80625	17	1½	·85625	18	1½	·90625	19	1½	·95625
16	1¾	·80729167	17	1¾	·85729167	18	1¾	·90729167	19	1¾	·95729167
16	2	·80833333	17	2	·85833333	18	2	·90833333	19	2	·95833333
16	2¼	·809375	17	2¼	·859375	18	2¼	·909375	19	2¼	·959375
16	2½	·81041667	17	2½	·86041667	18	2½	·91041667	19	2½	·96041667
16	2¾	·81145833	17	2¾	·86145833	18	2¾	·91145833	19	2¾	·96145833
16	3	·8125	17	3	·8625	18	3	·9125	19	3	·9625
16	3¼	·81354167	17	3¼	·86354167	18	3¼	·91354167	19	3¼	·96354167
16	3½	·81458333	17	3½	·86458333	18	3½	·91458333	19	3½	·96458333
16	3¾	·815625	17	3¾	·865625	18	3¾	·915625	19	3¾	·965625
16	4	·81666667	17	4	·86666667	18	4	·91666667	19	4	·96666667
16	4¼	·81770833	17	4¼	·86770833	18	4¼	·91770833	19	4¼	·96770833
16	4½	·81875	17	4½	·86875	18	4½	·91875	19	4½	·96875
16	4¾	·81979167	17	4¾	·86979167	18	4¾	·91979167	19	4¾	·96979167
16	5	·82083333	17	5	·87083333	18	5	·92083333	19	5	·97083333
16	5¼	·821875	17	5¼	·871875	18	5¼	·921875	19	5¼	·971875
16	5½	·82291667	17	5½	·87291667	18	5½	·92291667	19	5½	·97291667
16	5¾	·82395833	17	5¾	·87395833	18	5¾	·92395833	19	5¾	·97395833
16	6	·825	17	6	·875	18	6	·925	19	6	·975
16	6¼	·82604167	17	6¼	·87604167	18	6¼	·92604167	19	6¼	·97604167
16	6½	·82708333	17	6½	·87708333	18	6½	·92708333	19	6½	·97708333
16	6¾	·828125	17	6¾	·878125	18	6¾	·928125	19	6¾	·978125
16	7	·82916667	17	7	·87916667	18	7	·92916667	19	7	·97916667
16	7¼	·83020833	17	7¼	·88020833	18	7¼	·93020833	19	7¼	·98020833
16	7½	·83125	17	7½	·88125	18	7½	·93125	19	7½	·98125
16	7¾	·83229167	17	7¾	·88229167	18	7¾	·93229167	19	7¾	·98229167
16	8	·83333333	17	8	·88333333	18	8	·93333333	19	8	·98333333
16	8¼	·834375	17	8¼	·884375	18	8¼	·934375	19	8¼	·984375
16	8½	·83541667	17	8½	·88541667	18	8½	·93541667	19	8½	·98541667
16	8¾	·83645833	17	8¾	·88645833	18	8¾	·93645833	19	8¾	·98645833
16	9	·8375	17	9	·8875	18	9	·9375	19	9	·9875
16	9¼	·83854167	17	9¼	·88854167	18	9¼	·93854167	19	9¼	·98854167
16	9½	·83958333	17	9½	·88958333	18	9½	·93958333	19	9½	·98958333
16	9¾	·840625	17	9¾	·890625	18	9¾	·940625	19	9¾	·990625
16	10	·84166667	17	10	·89166667	18	10	·94166667	19	10	·99166667
16	10¼	·84270833	17	10¼	·89270833	18	10¼	·94270833	19	10¼	·99270833
16	10½	·84375	17	10½	·89375	18	10½	·94375	19	10½	·99375
16	10¾	·84479167	17	10¾	·89479167	18	10¾	·94479167	19	10¾	·99479167
16	11	·84583333	17	11	·89583333	18	11	·94583333	19	11	·99583333
16	11¼	·846875	17	11¼	·896875	18	11¼	·946875	19	11¼	·996875
16	11½	·84791667	17	11½	·89791667	18	11½	·94791667	19	11½	·99791667
16	11¾	·84895833	17	11¾	·89895833	18	11¾	·94895833	19	11¾	·99895833
17	0	·85	18	0	·9	19	0	·95	20	0	·1

PART I.

DECIMALS.

1. THE great advantage of the decimal notation is so obvious, that it is surprising this mode of division is not universally adopted for all measures of quantity, whether weight, time, or space. The inconvenience of a gradual transition to the new method would be scarcely felt; for a decimal coinage might be established, so far as regards money of account, by merely dividing the pound sterling into 1000 farthings instead of 960, as at present.

In the meantime the accountant is recommended to use the accompanying table, which contains the decimals for every farthing in the pound, in all calculations in which money fractions occur.

A brief outline of the principal operations in decimal arithmetic will be sufficient to illustrate its use; there are many popular treatises which contain full information on this branch of the science.

In expressing any number which has been counted, by a series of figures, as, two hundred and fifty-three (253), we use what is called the decimal scale of notation; for it is conventionally determined that the first figure on the right hand shall represent units, the second tens, the third hundreds, the fourth thousands, &c.; each digit acquiring a local value according to its position, by the following law—"Each figure shall increase tenfold in value with every remove toward the left." Thus we express *whole* numbers; but by an extension of the system, it is found that fractions, or *parts* of unity, may be expressed on the same scale, provided they be any multiple of ten, by determining that each digit shall *diminish* in value tenfold with every remove toward the right. A dot, called "the decimal point," is inserted to determine where whole numbers end and fractions begin.

If we wish to figure in decimals the expression, two hundred and fifty-three and one-half, instead of dividing unity into two parts and taking one, we divide it into *ten* equal parts and take five of them; but instead of writing $253\frac{5}{10}$, $253 \cdot 5$ will be sufficient; for we have determined that the denominator of the first fraction shall be one-tenth of the unit on its left, the second one-hundredth, the third one-thousandth, &c., *i.e.*, every successive digit shall be the tenth part or parts of that on its left.

The nominator of a fraction notes how many of the parts into which unity is divided shall be taken, and the *de*nominator the number of those parts. The expression: "divide unity into three equal parts and take two of them," is thus figured, $\frac{2}{3}$ (two-thirds), in which 2 is termed the nominator, and 3 the denominator. If we purpose to use *tenths* of unity instead of *thirds*, we reduce what is called the "vulgar fraction" to a decimal; the question is, How many tenths in two-thirds of one? By a rule hereafter to be explained the equation is thus stated:

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 10 \\ \frac{2}{3} \end{matrix}\right.,$$

in which ten units are to be counted by two-thirds.

It is obvious that ten times as many parts as 2 must be taken out of the 3;

$$\frac{2}{3} \times 10 = \frac{20}{3},$$

$$\text{which is } \frac{6}{10} \text{ and } \frac{2}{3} \text{ over;}$$

and so on,

$$\frac{6}{10} + \frac{6}{100} + \frac{6}{1000} + \frac{6}{10{,}000} + \text{\&c.},$$

ad infinitum; proving that although we can never reduce the fraction $\frac{2}{3}$ to an exactly equal decimal, we can find a decimal as near the value as we please; for instance, the

above $\cdot 666\dot{6}$, which is less than $\frac{2}{3}$ by not so much as $\frac{1}{10,000}$th part of unity. Thus, either entirely, or except a difference infinitely small, all fractions may be reduced to decimals.

2°. The terms, "addition," "subtraction," "multiplication," and "division," are sufficiently precise for the ordinary operations of arithmetic when confined to whole numbers; but when we apply the simple rules designated by these names to the higher relations of number and quantity, the terms prove inadequate, and are liable to confuse the mind. Before commencing these operations on fractional, but especially on positive and negative quantities, it would be well for the student either to extend his notions of their meaning, or to substitute more general and accurate expressions.

Thus, instead of *add* such and such quantities, we should rather say *collect* them; the process does not necessarily involve the idea of *increase*. Neither does subtraction always imply diminution; for if we subtract a debt from any one who owes it, we leave him a gainer by the amount of that debt. Multiplication is the *counting* one number by another; *i.e.*, taking that number as many times, or parts of times, as there are units or parts of units in the other. The result, therefore, is increase or diminution, according to the nature of the terms. Division is the measuring one quantity by another, which may be done, whether the divisor be greater or less than the dividend.

Decimals are added and subtracted in the same manner as whole numbers; taking care so to place the decimal points that they may fall under one another, as in the following examples:—

58·034	69·368
·51	58·653
2·361	10·715
100·1	
161·005	

3°. In multiplying decimals, proceed as in whole numbers; point off from the quotient as many decimal places as

are contained in the multiplier and multiplicand together When a limited number only of decimal places are required, the process may be abbreviated, by what is called the "contracted method." The *order* in which any number or quantity is counted by any other number or quantity, does not affect the result, therefore.

Invert the multiplier, proceed as before with the unit's digit, and cut off one figure from the right of the multiplicand, with every successive digit, remembering to carry the tens from the omitted figures. Place a dot over each omitted figure except the last. In order to find the place of the decimal point in the quotient, add as many ciphers, or dots, to the right of the quotient as there are dots in the multiplicand, and point off as before.

$$\begin{array}{r} 613{\cdot}2 \times {\cdot}35 \\ {\cdot}35 \\ \hline 30660 \\ 18396 \\ \hline 214{\cdot}620 \end{array} \qquad\qquad \begin{array}{r} 437{\cdot}\dot{8}\dot{5}\dot{6}1 \times 23{\cdot}46 \\ 64{\cdot}32 \\ \hline 8757122 \\ 1313568 \\ 175142 \\ 26271 \\ \hline 10272{\cdot}104\cdots \end{array}$$

4°. The common rule for the division of decimals is—"Make the divisor an integer by removing the decimal point to the end of it; remove, likewise, the decimal point in the dividend an equal number of places toward the right hand; supply any deficiency with ciphers on the right; then divide, as in whole numbers, and the integers in the dividend, thus altered, will give the integral part of the quotient; the decimal figures, if any, remaining in the dividend will give the same number of decimals in the quotient. Equalize the decimal places in the quotient, if necessary, by ciphers on the left."

$$\begin{array}{rcrcl} 295{\cdot}75 & \div & 8{\cdot}45 & = & 35 \\ 2 & \div & {\cdot}008 & = & 250 \\ 7202{\cdot}93 & \div & {\cdot}14 & = & 51449{\cdot}5 \\ \\ 45{\cdot}3884 & \div & {\cdot}0466 & = & 974 \\ {\cdot}83 & \div & {\cdot}03 & = & 27{\cdot}66\dot{6} \\ {\cdot}1530 & \div & 182{\cdot}6 & = & {\cdot}000837 \end{array}$$

It is sometimes useful to find the place of the decimal point *before* division. There are several methods of doing this; but perhaps the readiest is that of comparing the characteristics. This mode is the more desirable since a knowledge of it is indispensable in using logarithms. The principle will be explained in paragraph 6°.

LOGARITHMS.

5°. If we multiply any number, as (x), by itself, the product is called the *square* of x, and is thus algebraically expressed, x^2, in which 2 is called the *exponent* of the number represented by x. The exponent 2 denotes x multiplied *once* by itself, which also is termed the *second* power of x; x^3 is the cube or third power, x^4 the fourth power, &c. The student is here cautioned against falling into the common error of beginners; namely, that $x \times n$ times by itself is the nth power of x. Observe, $x \times x = x^2$, which is the *second*, that is the $1 + 1$th power of x; therefore $x \times n$ times by itself is the $n + 1$th power of x, or x^{n+1}.

Now, if x^{1+1} be the *second* power of x, the *first* power is x^1, written simply x, and x^{1-1} equals x^0. We shall hereafter find that x^0 signifies $\frac{x}{x}$, which equals unity, whatever number x may represent. This symbol, x^0, we may take as the zero point or unit of a table, which shall contain the powers of x, say to the 100th power. The symbols x^0, x^2, x^3, x^4, &c., might be arranged in one column, and opposite the numbers produced by these powers. Or rather, let us write in one column the numbers 1, 2, 3, &c., as far as we please, and opposite the powers of x which produce or come the nearest to these numbers. This would constitute a table of logarithms, of which x, whatever number it be, is called the base. This base does not appear in the table, but instead, the number represented by its exponent, namely, the logarithm.

In the published tables the number *ten* is usually selected as the base; in those of Babbage, for instance, which are the most convenient for the actuary and accountant, and which contain the logarithms of all numbers from 1 to 108,000, with ample directions for their use. These direc

tions state, that to multiply one number by another, we are to add their respective logarithms; thus $x^2 \times x^3 = x^5$, the number opposite the logarithm represented by 5 will be the product. On the same principle, to divide one number by another, subtract the logarithm of the divisor from that of the dividend. We can now more fully explain the symbol, x^0, the commencement of the system. Since x^4, divided by $x^2 = x^{4-2}$, or x^2; $x^2 \div x^2 = x^{2-2}$, or x^0. Let 5 be the number corresponding with the logarithm of which 2 is the exponent; then $\frac{5}{5} = 1$, the log of which is 0. We thus arrive at the conclusion above-mentioned, that, whatever number the base may be, the log of $1 = 0$.

Let us take 10 as the base of the system; what, then, is the log of 2? Now, there is no power of 10, whole or fractional, which, multiplied by itself, will produce 2; but a fraction may be found which will come sufficiently near to the required sum, namely, the following—$10^{\frac{3010300}{10000000}}$; that is, the log of $2 = \cdot 3010300$, the nearest decimal. In the same manner are found the logs of all other numbers.

6°. The log of 2, and, indeed, the logs of all numbers less than 10, must needs be fractional; what is termed the *characteristic* of such numbers is therefore a cipher prefixed to the log, thus 0·3010300, denoting that the number contains no figure in the ten's place. Accordingly, in the log of 10, the characteristic becomes 1, as 1·0000000; if the number consist of three figures it will be 2; and so on, always one less than the number of integral digits in the sum. The fractional part of the log is termed the *mantissa*.

But we require logs of numbers *less* than unity, as the fraction ·142, in which a digit is wanting in the unit's place. This deficiency is expressed by the negative characteristic, $\bar{1}$, which is read (minus one) thus, $\bar{1}\cdot 1522883$. Again, the decimal fraction, ·042, which is minus units, minus tenths, having no significant figure in either place, takes the characteristic $\bar{2}$, and so on, the *negative* characteristic being always one *more* than the number of ciphers prefixed to the decimal, as in the following table:

295·75	characteristic =	2
8·45	,,	0
7202·93	,,	3
·83	,,	$\bar{1}$
·03	,,	$\bar{2}$
·000837	,,	$\bar{4}$
182·6	,,	2

It is difficult for those unaccustomed to mathematical reasoning, to come at a clear conception of the relative values of positive and negative expressions; but the idea once acquired, problems containing these symbols are not by such complication the more difficult. Consider *positive* numbers as representing *gains* or possession, and *negative* as *debts* or *losses*. If a person possess £1, the unit is positive; if he lose it, we substitute a cipher to represent his financial condition. Here is no possession and no debt; but if in addition to losing £1 he should owe another, the cipher must be replaced by $\bar{1}$, the initial of a negative series of figures. If he possess £1, and at the same time owe £1, but his creditor should choose to cancel the obligation, we deduct the negative from the positive quantity $(1 - \bar{1})$. Now, to take away a loss is the same as to add an equivalent gain; therefore, change the sign of the negative quantity, and the equation becomes $(1 + 1) = 2$.

If a man have debts owing to one party amounting to £4, *i.e.*, $\bar{4}$, and at the same time another creditor absolve him from obligations to the amount of £11, we have the equation, $\bar{4} - \overline{11} = 7$; for the removal of the obligation to pay £11, when before he owed £15, leaves him a gainer to the amount of £7. The following equations will fully elucidate the principle:—

$$\bar{7} + 4 = \bar{3} \qquad \bar{3} + 1 = \bar{2}$$
$$\bar{7} + 12 = 5 \qquad 2 - \bar{2} = 4$$
$$1 - \bar{3} = 4 \qquad \bar{3} - 2 = \bar{5}$$
$$3 - 1 = 2 \qquad 2 - 2 = 0.$$

The rule then is—"To find the characteristic of the quotient, subtract the characteristic of the divisor from that of

the dividend, carrying *plus one* before subtracting, if the first two significant figures of the divisor be greater than those of the dividend."

Thus,

$$146{\cdot}08 \div {\cdot}00279 = \text{characteristic } 2 - \bar{3};$$

carry one, for 27 is greater than 14, and it becomes $2 - \bar{2} = 4$. The quotient must therefore have *five* integers before the decimal point.

$$\frac{295{\cdot}75}{8{\cdot}45} = 2 - 0,$$

carry one, for 84 is greater than 29, *i.e.*, $2 - 1 = 1$.

$$\left.\frac{2}{{\cdot}008}\right\} 0 - \bar{3} = 2; \qquad \left.\frac{7202{\cdot}93}{{\cdot}14}\right\} 3 - \bar{1} = 4.$$

THE CHAIN RULE.

7°. A method of stating and working arithmetical questions is much used on the Continent, which supersedes the great variety of rules usually taught in our schools. Almost every calculation that comes within the sphere of an accountant's business may, by its aid, be reduced to the form of a simple equation, and worked with great facility. Let us investigate the principles on which this method is founded.

Arithmetical calculation consists of the process of reasoning applied to questions in which numbers represent the relations of quantity. All who reason at all, whether learned or unlearned, can do so by only one and the same process. This process, in its various forms, is described by Aristotle and the logicians of his school, under the name of the syllogism. It is possible, however, that a more simple and general representation of the rational functions may yet be established, the result of clearer conceptions and a more profound analysis. Logic is not indeed an *invention*, as some imagine, neither is its instrumentality mainly serviceable for the discovery of *truth*, but *error*. It teaches how to avoid the various deceptions of the senses which tend to

pervert our intuitions, and to extricate a proposition from all ambiguity of expression or sophistical artifice.

So also in mathematics, we endeavour to divest a problem of all considerations not essential to its solution, and to show the intimate connection of the discoveries or speculations of science with the most obvious and familiar facts.

By way of illustration, let us take the following sentence, which in logic is called a proposition—"Every creature possessed of reason and liberty is accountable for his actions." Here we have an equation, the terms on one side being, creature, reason. liberty; and on the other the term accountability. As the two sides evidently balance each other, it may be compared to an equation involving unity, $a = a$, in which, if one side be measured or divided by the other, the result is $\frac{a}{a} = 1$. A series of three propositions having a mutual dependence is called an argument, or syllogism, thus:—

"Every creature possessed of reason and liberty is accountable for his actions.

"Man is a creature possessed of reason and liberty.

"Therefore, man is accountable for his actions."

Illustrating the maxim—"What can be predicated of the whole can be predicated of every part."

The syllogism is evidently a series of equations, the terms of which, had they consisted of numbers, and numbers always represent things, might have been arranged in two columns, putting any symbol as x, in the place of the question, whether man be accountable? or the conclusion which the argument is intended to verify.

$$\left.\begin{array}{r} x \\ \text{reason} \\ \text{creature} \\ \text{liberty} \end{array}\right\} = \left\{\begin{array}{l} \text{accountability} \\ \text{creature} \\ \text{man} \\ \text{accountability.} \end{array}\right.$$

If the above series were composed of numbers, we should multiply, *i. e.*, count the terms on the right and left respectively with each other, and then measure the product of the right column by that of the left, by which its agreement or disagreement with unity would be apparent.

Considered as a syllogism, the process is analogous; namely, compare the terms, observing at every step their connection and agreement with the middle term. See on which side

the balance preponderates, or whether the two sides neutralize each other, in which case the answer may be compared to unity. If the left column preponderate, the result may be represented numerically by a fraction $\frac{a}{b}$, of which the denominator b is greater than the nominator a.

These principles lead to the following general rule, which is applicable to all arithmetical questions:—

RULE.

Collect the terms by addition or subtraction, if necessary, and arrange them in two columns, commencing the left column with a symbol, say the letter x, of the result wanted, and place under it all the known or supposed terms. Commence the right column with a term equivalent to x, of the kind in which the answer is required. Under it place the unknown terms, opposite to similar terms on the left, and let the last term on the right column be of the kind in which the answer is required. Multiply all the terms in each column respectively by each other, divide the product of the right column by that of the left, the quotient will be the answer.

The only difficulty lies in the correct or logical statement of the terms, which a little practice with questions in various rules of arithmetic will soon enable the student to surmount.

The process admits of much abbreviation where vulgar fractions occur, by transposing the denominators to the opposite side, cancelling such numbers on each side of the equation as can be measured by a common divisor, and other expedients, which a very limited acquaintance with fractional arithmetic and the theory of equations will supply.

SIMPLE INTEREST.

8°. Simple Interest, or the amount paid for the use of a sum of money during the whole term of the loan, is easily computed when the principal, time, and rate per cent. consist of whole numbers. Thus, £100 for one year, at 5 per cent., amounts to £105. The interest of £100 for three years is $5 + 5 + 5$; this added to the principal £100, amounts to £115.

Cases, however, occur, in which one or more terms in the calculation consist of fractions, as when a sum composed of pounds, shillings, and pence, is lent at interest for a given number of days, not being an aliquot part of a year. The readiest way of computing such questions is, by reducing the several terms to the decimal parts of a year and of a pound. This is already done in tables I. and II., which contain respectively the decimals corresponding to every farthing in the pound, and to every day in the year. By the use of these tables, questions of interest may be calculated with a facility second only to that obtained by the use of the published interest tables, by merely counting the decimals corresponding to the given number of days, by those of the interest per pound. This gives the amount of interest for the given time, which, added to unity, and counted by the given sum, gives the whole amount of principal and interest for the given time. By using the interest of £1 instead of that of £100, the divisor is reduced to unity, and the operation thereby abbreviated.

£3, interest of £100 = $\frac{3}{100}$, or ·03 of £1

£3½, ,, ,, $\frac{3\frac{1}{2}}{100}$, or ·035 of £1

£4, ,, ,, $\frac{4}{100}$, or ·04 of £1

£5, ,, ,, $\frac{5}{100}$, or ·05 of £1

Examples.

What is the interest and amount of £100, in five years, at 5 per cent. per annum, simple interest?

Statement

What interest of	one hundred pounds?
If one hundred pounds in one year give	five pounds,
instead of one year,	five years;

Decimals corresponding with every Day in the Year.

Days.	Decimal.	Days.	Decimal.	Days.	Decimal.	Days.	Decimal.
1	·0027 3973	51	·1397 2603	101	·2767 1233	151	·4136 9863
2	·0054 7945	52	·1424 6575	102	·2794 5205	152	·4164 3836
3	·0082 1918	53	·1452 0548	103	·2821 9178	153	·4191 7808
4	·0109 5890	54	·1479 4521	104	·2849 3151	154	·4219 1781
5	·0136 9863	55	·1506 8493	105	·2876 7123	155	·4246 5753
6	·0164 3836	56	·1534 2466	106	·2904 1096	156	·4273 9726
7	·0191 7808	57	·1561 6438	107	·2931 5068	157	·4301 3699
8	·0219 1781	58	·1589 0411	108	·2958 9041	158	·4328 7671
9	·0246 5753	59	·1616 4384	109	·2986 3014	159	·4356 1644
10	·0273 9726	60	·1643 8356	110	·3013 6986	160	·4383 5616
11	·0301 3699	61	·1671 2329	111	·3041 0959	161	·4410 9589
12	·0328 7671	62	·1698 6301	112	·3068 4932	162	·4438 3562
13	·0356 1644	63	·1726 0274	113	·3095 8904	163	·4465 7534
14	·0383 5616	64	·1753 4247	114	·3123 2877	164	·4493 1507
15	·0410 9589	65	·1780 8219	115	·3150 6849	165	·4520 5479
16	·0438 3562	66	·1808 2192	116	·3178 0822	166	·4547 9452
17	·0465 7534	67	·1835 6164	117	·3205 4795	167	·4575 3425
18	·0493 1507	68	·1863 0137	118	·3232 8767	168	·4602 7397
19	·0520 5479	69	·1890 4110	119	·3260 2740	169	·4630 1370
20	·0547 9452	70	·1917 8082	120	·3287 6712	170	·4657 5342
21	·0575 3425	71	·1945 2055	121	·3315 0685	171	·4684 9315
22	·0602 7397	72	·1972 6027	122	·3342 4658	172	·4712 3288
23	·0630 1370	73	·2000 0000	123	·3369 8630	173	·4739 7260
24	·0657 5342	74	·2027 3973	124	·3397 2603	174	·4767 1233
25	·0684 9315	75	·2054 7945	125	·3424 6575	175	·4794 5205
26	·0712 3288	76	·2082 1918	126	·3452 0548	176	·4821 9178
27	·0739 7260	77	·2109 5890	127	·3479 4521	177	·4849 3151
28	·0767 1233	78	·2136 9863	128	·3506 8493	178	·4876 7123
29	·0794 5205	79	·2164 3836	129	·3534 2466	179	·4904 1096
30	·0821 9178	80	·2191 7808	130	·3561 6438	180	·4931 5068
31	·0849 3151	81	·2219 1781	131	·3589 0411	181	·4958 9041
32	·0876 7123	82	·2246 5753	132	·3616 4384	182	·4986 3014
33	·0904 1096	83	·2273 9726	133	·3643 8356	183	·5013 6986
34	·0931 5068	84	·2301 3699	134	·3671 2329	184	·5041 0959
35	·0958 9041	85	·2328 7671	135	·3698 6301	185	·5068 4932
36	·0986 3014	86	·2356 1644	136	·3726 0274	186	·5095 8904
37	·1013 6986	87	·2383 5616	137	·3753 4247	187	·5123 2877
38	·1041 0959	88	·2410 9589	138	·3780 8219	188	·5150 6849
39	·1068 4932	89	·2438 3562	139	·3808 2192	189	·5178 0822
40	·1095 8904	90	·2465 7534	140	·3835 6164	190	·5205 4795
41	·1123 2877	91	·2493 1507	141	·3863 0137	191	·5232 8767
42	·1150 6849	92	·2520 5479	142	·3890 4110	192	·5260 2740
43	·1178 0822	93	·2547 9452	143	·3917 8082	193	·5287 6712
44	·1205 4795	94	·2575 3425	144	·3945 2055	194	·5315 0685
45	·1232 8767	95	·2602 7397	145	·3972 6027	195	·5342 4658
46	·1260 2740	96	·2630 1370	146	·4000 0000	196	·5369 8630
47	·1287 6712	97	·2657 5342	147	·4027 3973	197	·5397 2603
48	·1315 0685	98	·2684 9315	148	·4054 7945	198	·5424 6575
49	·1342 4658	99	·2712 3288	149	·4082 1918	199	·5452 0548
50	·1369 8630	100	·2739 7260	150	·4109 5890	200	·5479 4521

Decimals corresponding with every Day in the Year, &c.

Days.	Decimal.	Days.	Decimal.	Days.	Decimal.	Days.	Decimal.
201	·5506 8493	251	·6876 7123	301	·8246 5753	351	·9616 4384
202	·5534 2466	252	·6904 1096	302	·8273 9726	352	·9643 8356
203	·5561 6438	253	·6931 5068	303	·8301 3699	353	·9671 2329
204	·5589 0411	254	·6958 9041	304	·8328 7671	354	·9698 6301
205	·5616 4384	255	·6986 3014	305	·8356 1644	355	·9726 0274
206	·5643 8356	256	·7013 6986	306	·8383 5616	356	·9753 4247
207	·5671 2329	257	·7041 0959	307	·8410 9589	357	·9780 8219
208	·5698 6301	258	·7068 4932	308	·8438 3562	358	·9808 2192
209	·5726 0274	259	·7095 8904	309	·8465 7534	359	·9835 6164
210	·5753 4247	260	·7123 2877	310	·8493 1507	360	·9863 0137
211	·5780 8219	261	·7150 6849	311	·8520 5479	361	·9890 4110
212	·5808 2192	262	·7178 0822	312	·8547 9452	362	·9917 8082
213	·5835 6164	263	·7205 4795	313	·8575 3425	363	·9945 2055
214	·5863 0137	264	·7232 8767	314	·8602 7397	364	·9972 6027
215	·5890 4110	265	·7260 2740	315	·8630 1370	365	1·0000 0000
216	·5917 8082	266	·7287 6712	316	·8657 5342	Year.	
217	·5945 2055	267	·7315 0685	317	·8684 9315	$\frac{1}{16}$	·062500
218	·5972 6027	268	·7342 4658	318	·8712 3288	$\frac{1}{12}$	·083333
219	·6000 0000	269	·7369 8630	319	·8739 7260	$\frac{1}{10}$	·100000
220	·6027 3973	270	·7397 2603	320	·8767 1233	$\frac{1}{8}$	·125000
221	·6054 7945	271	·7424 6575	321	·8794 5205	$\frac{1}{6}$	·166666
222	·6082 1918	272	·7452 0548	322	·8821 9178	$\frac{3}{16}$	·187500
223	·6109 5890	273	·7479 4521	323	·8849 3151	$\frac{1}{5}$	·200000
224	·6136 9863	274	·7506 8493	324	·8876 7123	$\frac{1}{4}$	·250000
225	·6164 3836	275	·7534 2466	325	·8904 1096	$\frac{3}{10}$	·300000
226	·6191 7808	276	·7561 6438	326	·8931 5068	$\frac{5}{16}$	·312500
227	·6219 1781	277	·7589 0411	327	·8958 9041	$\frac{1}{3}$	·333333
228	·6246 5753	278	·7616 4384	328	·8986 3014	$\frac{3}{8}$	·375000
229	·6273 9726	279	·7643 8356	329	·9013 6986	$\frac{2}{5}$	·400000
230	·6301 3699	280	·7671 2329	330	·9041 0959	$\frac{5}{12}$	·416666
231	·6328 7671	281	·7698 6301	331	·9068 4932	$\frac{7}{16}$	·437500
232	·6356 1644	282	·7726 0274	332	·9095 8904	$\frac{1}{2}$	·500000
233	·6383 5616	283	·7753 4247	333	·9123 2877	$\frac{9}{16}$	·562500
234	·6410 9589	284	·7780 8219	334	·9150 6849	$\frac{7}{12}$	·583333
235	·6438 3562	285	·7808 2192	335	·9178 0822	$\frac{3}{5}$	·600000
236	·6465 7534	286	·7835 6164	336	·9205 4795	$\frac{5}{8}$	·625000
237	·6493 1507	287	·7863 0137	337	·9232 8767	$\frac{2}{3}$	·666666
238	·6520 5479	288	·7890 4110	338	·9260 2740	$\frac{11}{16}$	·687500
239	·6547 9452	289	·7917 8082	339	·9287 6712	$\frac{7}{10}$	·700000
240	·6575 3425	290	·7945 2055	340	·9315 0685	$\frac{3}{4}$	·750000
241	·6602 7397	291	·7972 6027	341	·9342 4658	$\frac{4}{5}$	·800000
242	·6630 1370	292	·8000 0000	342	·9369 8630	$\frac{13}{16}$	·812500
243	·6657 5342	293	·8027 3973	343	·9397 2603	$\frac{5}{6}$	·833333
244	·6684 9315	294	·8054 7945	344	·9424 6575	$\frac{7}{8}$	·875000
245	·6712 3288	295	·8082 1918	345	·9452 0548	$\frac{9}{10}$	·900000
246	·6739 7260	296	·8109 5890	346	·9479 4521	$\frac{11}{12}$	·916666
247	·6767 1233	297	·8136 9863	347	·9506 8493	$\frac{15}{16}$	·937500
248	·6794 5205	298	·8164 3836	348	·9534 2466		
249	·6821 9178	299	·8191 7808	349	·9561 6438		
250	·6849 3151	300	·8219 1781	350	·9589 0411		

or,

$$\left.\begin{array}{r} x \\ 100 \\ 1 \end{array}\right\} = \left\{\begin{array}{r} 100 \\ 5 \\ 5 \end{array}\right\} = \pounds 25 + \overset{\text{Principal.}}{\pounds 100} = \pounds 125$$

Operation.

Cancelling 100 on each side of the equation, 5 + 5 = 25, which, added to 100 equals 125, the whole amount.

What is the amount of £100 in five years, if £150 produce £6 interest in one year?

$$\left.\begin{array}{r} x \\ 150 \\ 1 \end{array}\right\} = \left\{\begin{array}{r} 100 \\ 6 \\ 5 \end{array}\right\} = \overset{\text{Interest.}}{\pounds 20} + \overset{\text{Principal.}}{\pounds 100} = \pounds 120$$

Proof.

$$\left.\begin{array}{r} \text{What principal will produce} \\ \text{if interest £30 be produced by} \end{array}\right\} = \left\{\begin{array}{r} \text{20 interest} \\ \text{150 principal?} \end{array}\right\} = \pounds 100$$

What is the amount of £537 625, for five years, at 4 per cent. per annum?

$$\left.\begin{array}{r} x \\ 1 \\ 1\text{ yr.} \end{array}\right\} = \left\{\begin{array}{r} 537{\cdot}625 \\ {\cdot}04 \\ 5\text{ yrs.} \end{array}\right.$$

£107·525 Interest.
537·625 Principal.

645·15 Amount

By collecting the terms, (·04 × 5) + 1 = 1·2, the amount of £1 in one year, the question might be stated thus:—

$$\left.\begin{array}{r} x \\ 1 \end{array}\right\} \quad \left\{\begin{array}{r} 537{\cdot}625 \\ 1{\cdot}2 \end{array}\right\} = 645{\cdot}15, \text{ or } \pounds 645\ \ 3s.$$

Proof.

Interest of £1 in five years = (·04 × 5) or ·2;

$$\therefore \left.\begin{array}{r} x \\ 2 \end{array}\right\} \quad \left\{\begin{array}{l} 107{\cdot}525 \text{ Interest.} \\ 1 \end{array}\right.$$

537·625 Principal.

If a term occurs of a higher or lower denomination than that in which the answer is required, another connecting term will be necessary.

What is the interest and amount of £300, in four years, at fifty shillings per cent. per annum?

$$\left.\begin{array}{r} x \\ \pounds100 \\ 1 \text{ yr.} \\ \pounds1 \end{array}\right\} = \left\{\begin{array}{l} \pounds300 \\ 50s. \\ 4 \text{ yrs.} \\ \pounds1 \end{array}\right\} = \overset{\text{Interest.}}{30} + \overset{\text{Principal.}}{\pounds300} = \pounds330.$$

Proof.

$$\text{Interest in 4 yrs. of } \left.\begin{array}{r} x \\ 200s. \\ \pounds1 \end{array}\right\} \left\{\begin{array}{l} 30 \\ \pounds100 \\ 20s \end{array}\right\} = \pounds300.$$

Operation.

Cancelling the ciphers on each side, we have,

$$\frac{3 \times 5 \times 4 \times 1}{1 \times 2} = \frac{60}{2} = \overset{\text{Interest.}}{\pounds30} + \pounds300 = \pounds330 \text{ Amount.}$$

What is the interest on £6, for twenty days, at 5 per cent. per annum?

Select from Table II, the decimal of a year corresponding to twenty days, and state as before.

$$\left.\begin{array}{r} x \\ \pounds1 \\ 1 \text{ yr.} \end{array}\right\} = \left\{\begin{array}{r} \pounds6 \\ \cdot05 \\ \cdot05479 \end{array}\right\} \cdot16438, \text{ or } 3s.\ 3\tfrac{1}{4}d.$$

Proof.

$$\left.\begin{array}{r} x \\ \cdot05 \\ \cdot05479 \end{array}\right\} = \left\{\begin{array}{l} \cdot16438 \\ \pounds1 \\ 1 \text{ yr.} \end{array}\right\} = \pounds6.$$

9°. When the *principal* is the question, the sum, time, and rate per cent. being given, the proportion is inverse. *i. e*, the middle term is in inverse ratio to the first.

B returns A £645 3*s*., principal and interest for the loan of a sum for five years, at 4 per cent.; what was the sum advanced?

Statement

What principal in five years will amount to 645·15, if £1 in five years amount to 1·2?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 645{\cdot}15 \\ 1{\cdot}2 \end{matrix}\right.$$

The question concerns the principal, which involves the element of time; the *more* interest £1 produces in five years, the *less* principal will be required to produce the sum £645·15. The sum must be measured by what £1 will produce in five years. The above statement is therefore incorrect. It should stand thus:—

What principal in five years will amount to £645·15, if the amount 1·2 be produced in five years by £1?

$$\left.\begin{matrix} x \\ 1{\cdot}2 \end{matrix}\right\} = \left\{\begin{matrix} 645{\cdot}15 \\ 1 \end{matrix}\right\} = £537{\cdot}625.$$

10°. In how many years will £537 12*s*. 6*d*. amount to £645 3*s*., at 4 per cent., simple interest?

Since the amount equals principal + interest, the amount minus the principal will give the interest for the whole term.

645·15	Amount.
537·625	Principal.
107·525	Interest.

The question now is, in how many years will £537·625 produce £107·525 interest, if in one year ·04 interest is produced by £1. The larger the capital, the fewer the years to produce the given amount; the interest must therefore be measured by the capital.

$$\left.\begin{matrix} x \\ 537{\cdot}625 \\ {\cdot}04 \end{matrix}\right\} = \left\{\begin{matrix} 107{\cdot}525 \\ £1 \end{matrix}\right\} \left\{ \frac{107{\cdot}525}{537{\cdot}625 \times {\cdot}04} \right.$$

$$= \frac{107{\cdot}525}{21{\cdot}505} = 5 \text{ years.}$$

At what rate per cent. will £537 12*s*. 6*d*. amount to £645 3*s*. in five years?

645·15
51·625

The amount of interest is £107 525

What rate per pound, if £537·625 in five years produce £107·525 interest?

$$\left.\begin{array}{r} x \\ 537{\cdot}625 \\ 5 \text{ yrs.} \end{array}\right\} = \left\{\begin{array}{c} £1 \\ 107\ 525 \\ 1 \text{ yr.} \end{array}\right\} = \frac{107{\cdot}525}{2688{\cdot}125} = {\cdot}04.$$

11°. When logarithms are used, it may be convenient to take the rate *per cent.* instead of the rate *per pound*, according to the following rule; virtually, the process is the same.

To find the amount of interest on a sum of money at any given time. Add together the logs of the principal, the rate, and the time, and subtract 2 from the characteristic of the resulting logarithm, which, since it removes the decimal point two places toward the left, is the same as dividing by 100.

What is the interest on £537 12*s.* 6*d.*, in five years, at 4 per cent.?

$$\left.\begin{array}{r} x \\ 100 \\ 1 \text{ yr.} \end{array}\right\} = \left\{\begin{array}{ll} 537{\cdot}625 & = \log 2{\cdot}7304795 \\ 4 & = \log 0{\cdot}6020600 \\ 5 \text{ yrs.} & = \log 0{\cdot}6989700 \end{array}\right.$$

$$\begin{array}{l} 4{\cdot}0315095 \div 100 \\ 2 \\ \hline 2{\cdot}0315095 = £107{\cdot}525 \end{array}$$

Required the interest on £325 6*s.* 4*d.*, for seven years, six weeks, and three days, at £3 4*s.* 9*d.* per cent. per annum.

$$\left.\begin{array}{r} x \\ 100 \\ 1 \text{ yr.} \end{array}\right\} = \left\{\begin{array}{ll} 325{\cdot}3166 & = \log 2{\cdot}5123062 \\ 3{\cdot}2375 & = \log 0{\cdot}5102098 \\ 7{\cdot}1233 \text{ yrs.} & = \log 0{\cdot}8526812 \end{array}\right.$$

$$\begin{array}{l} 3{\cdot}8751972 \div 100 \\ 2 \\ \hline 1{\cdot}8751972 = 75{\cdot}0234, \end{array}$$

or £75 0*s.* 5½*d.*

12°. DISCOUNT is the allowance made for the payment of a sum of money before it becomes due. The sum paid after deducting the discount is called the present value.

What is the present value of £100, paid down now, instead of being paid at the end of one year, the rate of interest being 5 per cent.?

The commercial rule is—Deduct for the discount the whole amount of interest on the sum till it becomes due. Thus, 100 − 5, or £95, would be the sum received as the present value of £100. But £95 in one year will produce at the same rate, only £4 15*s.* interest. The commercial discount is, consequently, considerably *more* than the present value, which is £95 4*s.* 8$\frac{4}{7}$*d.*

It is usual for bankers to charge discount for three days more than the number of days the bill has to run; these are called "days of grace."

Since discount is merely interest deducted in advance, the formulæ for interest apply equally to questions of discount.

What is the discount on a bill due sixty-nine days hence, for £250, at 4 per cent.?

Statement

What discount on £250, if the discount per pound in one year be ·04, how much in ·1973 of a year?

$$\left.\begin{matrix} x \\ 1 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 250 \\ \cdot 04 \\ \cdot 1973 \end{matrix}\right\} = 1{\cdot}973, \text{ or } £1\ 19s.\ 5\tfrac{1}{4}d.$$

The *commercial* discount, we have said, is simply the deduction of interest for the time that elapses till the sum becomes due, and is found by sum × interest × time. But the *true* discount is the difference between the sum due at the end of the term, and the present value, or the sum that would produce the amount at the same rate of interest during the term.

What is the true discount on a bill due sixty-nine days hence, for £250, at 4 per cent.?

First find the present value, *i. e.*, what sum will produce £250 in seventy-two days, if ·04 interest is produced in one year by £1, in ·1973 of a year, what is the amount?

$$\left.\begin{array}{r} x \\ \left(\begin{array}{l}\cdot 01 \\ \cdot 1973\end{array}\right)+1\end{array}\right\} = \left\{\begin{array}{r} 250 \\ 1\end{array}\right\} = \frac{250}{1\cdot 0078} = £248\cdot 065,$$

the sum that will produce £250 in seventy-two days.

$$\begin{array}{r} 250\ 000 \\ 248\cdot 065 \\ \hline 1\cdot 935 \end{array}$$

1·935, or £1 18*s.* 8½*d.* true discount.

£1	19	5¼	Commercial discount.
1	18	8½	True discount
		8¾	Excess.

What is the present value of £100, due two years hence, at 5 per cent. per annum?

Statement.

What sum in two years will produce £100, if in two years £110 is the amount of £100?

$$\left.\begin{array}{r} x \\ 110\end{array}\right\} = \left\{\begin{array}{r} 100 \\ 100\end{array}\right\} = \frac{1000}{11} = £90\ 18s.\ 2\tfrac{8}{11}d.,$$ the present value.

GENERAL REMARKS ON DISCOUNTS AND PERCENTAGES.

13°. Suppose the cost of producing a parcel of goods be £75, and a profit of 25, *i.e.*, ¼ per cent., be required. £25 is added to £75, and the goods are sold for £100; but £25 is only ⅓ of £75.

Therefore, to produce a profit of ¼ per cent., ⅓ of the nett value must be added From this we obtain an universal rule for all fractions.

To take off $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{1}{21}$, $\frac{1}{22}$, $\frac{1}{99}$,

Add . . $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{19}$, $\frac{1}{20}$, $\frac{1}{21}$, $\frac{1}{98}$,

$$i.e., \text{ to take off } \frac{a}{n}, \text{ add } \frac{a}{n-1}.$$

From the above data we may also deduce another rule. For $\frac{1}{4}$ deduction we add $\frac{1}{3}$; now

$$\frac{1}{3} = \frac{1 \times 25}{3 \times 25} = \frac{25}{75}, \text{ and } \frac{25}{75+25} = \frac{25}{100}.$$

Therefore, to find how much must be added, to allow a given percentage from the gross price, take a fraction, the nominator being the required percentage, and the denomi nator 100 minus the nominator.

To take of $\frac{5}{100}$, $\frac{20}{100}$, $\frac{25}{100}$, $\frac{30}{100}$, $\frac{35}{100}$, $\frac{75}{100}$, $\frac{95}{100}$ per cent

Add . . $\frac{5}{95}$, $\frac{20}{80}$, $\frac{25}{75}$, $\frac{30}{70}$, $\frac{35}{65}$, $\frac{75}{25}$, $\frac{95}{5}$,

Or . . $\frac{1}{19}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{7}$, $\frac{7}{13}$, $\frac{3}{1}$ or 3, 19.

$$i.e, \text{ to take off } \frac{n}{100}, \text{ add } \frac{n}{100-n}.$$

What must be added to the nett to allow $22\frac{1}{2}$ per cent. from the gross?

Nett price 12*s.* 11*d.* Add 12*s.* 11*d.* $\div \frac{22\frac{1}{2}}{77\frac{1}{2}} = \frac{45}{155} = \frac{9}{31}$,

and 12*s.* 11*d.* $\times$ 9 $\div$ 31 = 3*s.* 9*d.* + 12*s.* 11*d.* = 16*s.* 8*d.* gross price.

A gives £100 for a parcel of goods, with a reduction of 20 per cent. discount, or £80 nett. B gives £100 for a similar parcel, with a reduction of 30 per cent., or £70 nett. For the same article A gives £80, B £70. The question is—How much cheaper does B buy than A? How much less is £70 than £80? It is by the *larger* sum that the difference must be measured; because you wish to know, how much per cent. the smaller sum is *less* than the larger one. Ten pounds, the difference, is $\frac{1}{8}$ of £80; it is $\frac{1}{8}$ less, and $\frac{1}{8}$ of 100, $\frac{100}{8}$, is $12\frac{1}{2}$ per cent.

If, however, the question had been reversed and placed thus: How much dearer did A pay for his goods than B? In this case, it is with the *smaller* sum that the difference must be measured; you have to determine how much per cent. *more* £80 is than £70. The difference, £10, is $\frac{1}{7}$ of £70 and $\frac{1}{7}$ of £100, or $\frac{100}{7}$, is $14\frac{2}{7}$, *i.e.*, $14\frac{2}{7}$ per cent. *less.*

In this supposed transaction, therefore, B purchases his goods $12\frac{1}{2}$ per cent. *lower* than A; but A paid $14\frac{2}{7}$ per cent. *higher* than B.

In numerous ways, in measuring the difference of sums, numbers, or quantities, by percentages, it is extremely important to bear this distinction clearly in mind. It makes all the difference whether we compare the *larger* with the *smaller* amount, or the *smaller* with the *larger* amount.

Suppose A and B hold railway stock; A paid £100 per share, and B had bought the same at £50 per share. The price that A paid was double of that which B paid, consequently 100 per cent. *dearer*. The price which B paid was $\frac{1}{2}$ of that which A paid, therefore 50 per cent. *cheaper*.

There is another very common mistake into which traders fall with respect to discounts. A person finds by observation that the expenses of conducting his business form altogether 10 per cent. on his returns. He thinks himself entitled to 10 per cent. profit. The two together make 20 per cent. He adds $\frac{1}{5}$, or 20 per cent., to the cost of every article. At the end of the year, supposing the estimate of his expenses to be right, what will be the profit on his returns? In the place of 10 per cent., as he expected, he will find it to be $6\frac{2}{3}$ per cent. Take an example:—

A person pays £100 for a parcel of goods. He adds $\frac{1}{5}$, or 20 per cent., and sells them for £120. His returns at the end of the year are made up in this way: The expenses of his business are equal to 10 per cent. on his returns. On this transaction 10 per cent. is £12, reducing the nett sum received to £108, or 8 per cent; which on £120, the amount of his returns, is only $6\frac{2}{3}$ per cent.

Example.

How much of £100 is £8 of £120? or, what percentage will £100 make if £120 make £8?

The *greater* the capital the *less* the percentage; therefore

$$\left.\begin{matrix} x \\ 120 \end{matrix}\right\} = \left\{\begin{matrix} 100 \\ 8 \end{matrix}\right\} = \frac{8 \times 10}{12} \text{ or } \frac{80}{12} = 6\tfrac{2}{3}.$$

In like manner, if a person return £30,000, having put on 20 per cent. on the cost of each article, the prime cost of the whole of his goods must have been 30,000 − 5000, or £25,000. Well, from £30,000 he has 10 per cent., or £3000, to deduct for charges, leaving his nett receipts £27,000, or £2000 above the first cost, which on £30,000, the amount of his returns, is a profit of $6\frac{2}{3}$ per cent.; while all the time he had expected, as his returns mounted up during the year, that 10 per cent. was clear profit. Thus, after having done business to the extent of £30,000, he is puzzled when his stock-taking shows his profits to be only £2000.

Illustrations

14°. What profit, at £3 7*s*. 6*d* per cent., is there on a capital of £2044 3*s*. 6*d*.?

$$\left.\begin{matrix} x \\ 100 \end{matrix}\right\} = \left\{\begin{matrix} 2044{\cdot}175 \\ 3{\cdot}375 \end{matrix}\right\} 68{\cdot}9909, \text{ or } £68\ 19s.\ 9\tfrac{3}{4}d.$$

Proof.

How much per cent., if the profit on £2044·175 be 68·9909?

$$\left.\begin{matrix} x \\ 2044{\cdot}175 \end{matrix}\right\} = \left\{\begin{matrix} 100 \\ 68{\cdot}9909 \end{matrix}\right\} 3{\cdot}375, \text{ or } £3\ 7s.\ 6d.$$

What is the quarterly interest of £96 7*s*. 6*d*., at 4 per cent.?

$$\left.\begin{matrix} x \\ 1 \\ 1 \text{ yr.} \end{matrix}\right\} = \left\{\begin{matrix} 96{\cdot}375 \\ {\cdot}04 \\ \tfrac{1}{4} \text{ yr.} \end{matrix}\right\} = {\cdot}96375, \text{ or } 19s\ 3\tfrac{1}{4}d$$

Proof.

What rate per pound, if 96·375 in $\frac{1}{4}$ year produce ·96375?

$$\left.\begin{matrix} x \\ 96{\cdot}375 \\ \frac{1}{4} \end{matrix}\right\} = \left\{\begin{matrix} 1 & \\ & {\cdot}96375 \\ 1 & \end{matrix}\right\} \frac{{\cdot}96375}{24{\cdot}094} = {\cdot}04.$$

The cost of producing an article is 28*s.* per cwt., which is sold at 38*s.* How much is that per cent.?

Statement.

How much of £100 is 10*s.* of £1 18*s.*?
Or, What profit will £100 produce if £1 18*s.* produce 10*s.*?

$$\left.\begin{matrix} x \\ 1{\cdot}9 \end{matrix}\right\} = \left\{\begin{matrix} 100 & \\ & {\cdot}5 \end{matrix}\right\} = 26{\cdot}3105, \text{ or } £26\ 6s.\ 2\tfrac{1}{2}d. \text{ per cent.}$$

If 8 cwt. 1 qr. 24 lbs. cost £26 4*s.* 5*d.*, how much is that per cwt.?

$$1 \text{ qr. } 24 \text{ lbs.} = 52 \text{ lbs., which is } \frac{52}{112}, \text{ or } {\cdot}4643, \quad \therefore$$

$$\left.\begin{matrix} x \\ 8{\cdot}4643 \end{matrix}\right\} = \left\{\begin{matrix} 1 \text{ cwt.} \\ 26{\cdot}221 \end{matrix}\right\} 3{\cdot}0978, \text{ or } 61s.\ 11\tfrac{1}{2}d. \text{ per cwt.}$$

ANNUITIES AT SIMPLE INTEREST.

15°. An annuity is any periodical income payable at equal intervals; as yearly, half yearly, quarterly, &c.

Annuities to continue a fixed number of years are called "annuities *certain*;" those which are to be paid only so long as one or more individuals shall live, are called "*contingent*, or life annuities." An annuity to continue for ever is called a "perpetuity." When the first payment of an annuity is not to commence till the expiration of a given time, it is called a "*reversionary*, or deferred annuity."

Example.

What annuity, payable yearly, may be purchased for £2360, at 4 per cent. simple interest?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 2360 & \\ & \cdot 04 \end{matrix}\right\} = 94{\cdot}4, \text{ or } £94\ 8s.$$

ANNUITIES FORBORNE.

16°. When the payment of an annuity has been forborne for a certain number of years, the sum of all the payments, together with the sum of the interest of each payment, from the period of its becoming due till the payment of the whole, will be the amount for the lapsed time.

Suppose an annuity of £20 per annum were forborne five years, the rate of interest being 5 per cent., *i. e.*, £1 per annum for the £20, what is the amount due at the end of the five years?

The last payment, being received at the time it falls due, is simply

£20.

The last payment but one, being due a year ago, is

20 + 1 (1 year's interest).

The last payment but two, being due two years ago, is

20 + 1 × 2 years' interest.

The last payment but three, being due three years ago, is

20 + 1 × 3 years' interest.

The last payment but four, being due four years ago, is

20 + 1 × 4 years' interest.

The last payment but four being the first payment of the annuity, and due at the *end* of the first year, has 5 — 1 year's interest due thereon. The amount, then, consists of a series of terms in arithmetical progression, whose common difference is one It is therefore amenable to the following laws:

"Any term of an equidifferent or arithmetical series, is equal to the first term, increased by the common difference, multiplied by one less than the number of terms."

"The sum of the terms of an equidifferent series is equal to the sum of the first and last terms, multiplied by half the number of terms."

First term £20
Last term 20 + 4

44
$2\frac{1}{2}$ half the number of terms.

88
22

£110 sum of five terms.

As stated above, the sum of the terms contains the annuity, counted by the number of years, together with interest successively for the number of years less one. If we, then, multiply the number of years by the number of years less one, and count the interest by that product, which is nearly the square of the number of years, we get twice the interest of £1, or the interest of £2 for the whole term, which, divided by 2, gives the interest of £1 for the whole term; to this add £1, multiplied by the number of years for the whole amount of an annuity of £1 for the lapsed time, which must be counted by the given annuity.

Questions of this kind involve an infinite series, for the summation of which a general formula and explanation may be found in most of the works on algebra; to which the student is referred for a satisfactory demonstration. Meanwhile we deduce a general formula in accordance with our system, which, for all practical purposes, will prove sufficient.

Statement.

What interest on £1 per annum if £1 forborne five years

equals $\frac{5 \times 4 \times \cdot 05}{2}$,

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\frac{\overset{1}{5 \times 4 \times \cdot 05}}{2}\right\} = \begin{array}{r} \cdot 5 \\ +\ 5 \\ \hline 5\cdot 5 \\ 20 \\ \hline 110\ 00 \end{array} \begin{array}{l} \text{amount of interest.} \\ \\ \\ \text{amount of £1 annuity.} \\ \\ \\ \text{amount due at the end of five years.} \end{array}$$

17°. What is the amount of an annuity of £436 forborne twelve years, at $3\frac{1}{2}$ per cent. simple interest?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\frac{\overset{\text{£1 per an.}}{12 \times 11 \times \cdot 035}}{2} + 12 \times 436.\right.$$

Operation.

$$\begin{array}{rl}
12 & \\
11 & \\
\hline
132 & \\
035 & \\
\hline
660 & \\
396 & \\
\hline
2)\ 4\cdot 620 & \\
\hline
2\cdot 310 & \\
+12 & \\
\hline
14\cdot 310 & \left\{\begin{array}{l}\text{Amount of annuity of £1,} \\ \text{forborne 12 years.}\end{array}\right. \\
634 & \text{Annuity (inverted).} \\
\hline
57240 & \\
4293 & \\
858 & \\
\hline
6239\cdot 1 & \text{or £6239 3s. } 2\tfrac{1}{4}d.
\end{array}$$

If an annuity forborne twelve years, at $3\frac{1}{2}$ per cent., amount to £6239·16, what was the annuity?

By last example, annuity of £1, forborne twelve years, amounts to £14·31,

$$\therefore \left.\begin{matrix} x \\ 14{\cdot}31 \end{matrix}\right\} = \left\{\begin{matrix} 6239{\cdot}16 \\ 1 \end{matrix}\right\} = £436.$$

For the methods of finding the number of years and rate per cent. from the remaining data, see Jones's work on the "Value of Annuities."

PRESENT VALUE OF ANNUITIES AT SIMPLE INTEREST.

18°. Annuities being usually calculated at compound interest, tables are constructed for finding the present value. The following is the process for annuities at simple interest.

What is the present value of £1 per annum for three years, at 3 per cent.?

This question involves a series, for the summation of which no general formula has yet been discovered. The present value for any term of years may, however, be found, in accordance with our method, in a series of equations, adding together the results.

Statement.

What is the present value of £1 per annum if $1 + (1 \times {\cdot}03)$ is its amount in one year?

$$\left.\begin{matrix} x \\ 1{\cdot}03 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 1 \end{matrix}\right\} = \frac{1}{1{\cdot}03} = {\cdot}97087$$

$$\left.\begin{matrix} x \\ 1 + (2 \times {\cdot}03) \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 1 \end{matrix}\right\} = \frac{1}{1{\cdot}06} = {\cdot}94339.$$

$$\left.\begin{matrix} x \\ 1 + (3 \times {\cdot}03) \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 1 \end{matrix}\right\} = \frac{1}{1{\cdot}09} = \frac{{\cdot}91743}{2{\cdot}8317}, \text{ or } £2\ 16s.\ 8d.$$

The reciprocal of any number is unity divided by that

Amount of £1 at Compound Interest, in any Number of Years not exceeding fifty.

Years.	3 per Cent.	4 per Cent.	5 per Cent.
1	1·0300 0000	1·0400 0000	1·0500 0000
2	1·0609 0000	1·0816 0000	1·1025 0000
3	1·0927 2700	1·1248 6400	1·1576 2500
4	1·1255 0881	1·1698 5856	1·2155 0625
5	1·1592 7407	1·2166 5290	1·2762 8156
6	1·1940 5230	1·2653 1902	1·3400 9564
7	1·2298 7387	1·3159 3178	1·4071 0042
8	1·2667 7008	1·3685 6905	1·4774 5544
9	1·3047 7318	1·4233 1181	1·5513 2822
10	1·3439 1638	1·4802 4428	1·6288 9463
11	1·3842 3387	1·5394 5406	1·7103 3936
12	1·4257 6089	1·6010 3222	1·7958 5633
13	1·4685 3371	1·6650 7351	1·8856 4914
14	1·5125 8972	1·7316 7645	1·9799 3160
15	1·5579 6742	1·8009 4351	2·0789 2818
16	1·6047 0644	1·8729 8125	2·1828 7459
17	1·6528 4763	1·9479 0050	2·2920 1832
18	1·7024 3306	2·0258 1652	2·4066 1923
19	1·7535 0605	2·1068 4918	2·5269 5020
20	1·8061 1123	2·1911 2314	2·6532 9771
21	1·8602 9457	2·2787 6807	2·7859 6259
22	1·9161 0341	2·3699 1879	2·9252 6072
23	1·9735 8651	2·4647 1555	3·0715 2376
24	2·0327 9411	2·5633 0417	3·2250 9994
25	2 0937 7793	2·6658 3633	3·3863 5494
26	2·1565 9127	2·7724 6979	3·5556 7269
27	2·2212 8901	2·8833 6858	3·7334 5632
28	2·2879 2768	2·9987 0332	3·9201 2914
29	2·3565 6551	3·1186 5145	4·1161 3560
30	2·4272 6247	3·2433 9751	4·3219 4238
31	2·5000 8035	3·3731 3341	4·5380 3949
32	2·5750 8276	3·5080 5875	4·7649 4147
33	2·6523 3524	3·6483 8110	5·0031 8854
34	2·7319 0530	3·7943 1634	5·2533 4797
35	2·8138 6245	3·9460 8899	5·5160 1537
36	2·8982 7833	4·1039 3255	5·7918 1614
37	2·9852 2668	4·2680 8986	6·0814 0694
38	3·0747 8348	4·4388 1345	6·3854 7729
39	3·1670 2698	4·6163 6599	6·7047 5115
40	3·2620 3779	4·8010 2063	7·0399 8871
41	3·3598 9893	4·9930 6145	7·3919 8815
42	3·4606 9589	5·1927 8391	7·7615 8755
43	3·5645 1677	5·4004 9527	8·1496 6693
44	3·6714 5227	5·6165 1508	8·5571 5028
45	3·7815 9584	5·8411 7568	8·9850 0779
46	3·8950 4372	6·0748 2271	9·4342 5818
47	4·0118 9503	6·3178 1562	9·9059 7109
48	4·1322 5188	6·5705 2824	10·4012 6965
49	4·2562 1944	6·8333 4937	10·9213 3313
50	4·3839 0602	7·1066 8335	11·4673 9978

number; therefore, to find the present value of £1 per annum for any number of years, we add together the reciprocals of the amounts at the end of each year. This product, counted by the given annuity, will be its present value for the given term of years.

COMPOUND INTEREST.

19°. When the interest of money is added to the principal, at periodical intervals, the interest accumulating on this increasing capital is called "compound interest."

The following series exhibits the law of increase when the interest is payable yearly, on £100, at 5 per cent.

Amount at the end of the first year, 100 + 5.
,, ,, second year, 100 + £10 5*s.*
,, ,, third year, 100 + £15 15*s.* 3*d.*

Since 1 + ·05 is the ratio, or amount of £1 with interest at the end of the first year, the amount of any other sum in one year will be in the same proportion; *i. e.*, as 1 is to 1·05, so is any sum to its amount in one year. But 1·05 forms a new principal, the interest of which, with the principal, gives the amount of £1, the original principal, at the end of the second year; therefore,

$$1 : 1 + \cdot05 :: 1 + \cdot05,$$

or, what is the amount in one year of 1 + ·05, if £1 in one year produces 1 + ·05?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 1 + \cdot05 \\ 1 + \cdot05 \end{matrix}\right\} = (1 + \cdot05)^2.$$

The square of the ratio 1 + ·05 is, then, the fourth term of the proportion : 1 : 1 + ·05 :: 1 + ·05 : $(1 + \cdot05)^2$, or the amount of £1 at the end of the *second* year

To find the amount at the end of the *third* year, another statement will be required:

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} (1 + \cdot05)^2 \\ 1 + \cdot05 \end{matrix}\right\} = (1 + \cdot05)^3.$$

And so on, to twenty or any number of years, making a fresh equation for every step in the progression, until we

arrive at the last term; which, multiplied by the given sum, will give the whole amount of any sum in a given number of years. This method, too tedious for practical use, is much shortened by the aid of logarithms, according to the following general rule:—

The amount of £1, at compound interest, for any number of equal intervals, is its ratio, or amount with interest for the first interval, raised to the power indicated by the number of equal intervals. In twenty years, $1 + \cdot 05$ amounts to $(1 + \cdot 05)^{20}$, the twentieth power of $1 + \cdot 05$. The log of 1·05, multiplied by 20, equals the log of the amount of £1 in 20 years. On this principle Table III. is constructed, which gives the amount of £1 at compound interest, at various rates per cent., from one to fifty years inclusive. We have only to select from this table the term opposite to the given number of years, and multiply it by the given amount.

Questions of which the data are not contained in this table may easily be solved by the aid of logarithms and the decimal table.

The following table, which contains the logarithms of the *ratios*, or amount of £1 for the first term or interval, at various rates per cent., will be found useful in calculations of compound interest.

Rate of Interest.	Logarithm of Ratio.	Rate of Interest.	Logarithm of Ratio.
1	·0043214	5½	·0232525
1¼	·0053950	5¾	·0242804
1½	·0064660	6	·0253059
1¾	·0075344	6¼	·0263289
2	·0086002	6½	·0273496
2¼	·0096633	6¾	·0283679
2½	·0107239	7	·0293838
2¾	·0117818	7¼	·0303973
3	·0128372	7½	·0314085
3¼	·0138901	7¾	·0324173
3½	·0149403	8	·0334238
3¾	·0159881	8¼	·0344279
4	·0170333	8½	·0354297
4¼	·0180761	8¾	·0364293
4½	·0191163	9	·0374265
4¾	·0201540	9¼	·0384214
5	·0211893	9½	·0394141
5¼	·0222221	9¾	·0404045

Examples.

20°. What is the amount of £325 6*s.* 4*d.* in seven years, six weeks, and three days, at £3 4*s.* 9*d.* per cent. compound interest, convertible yearly?

$$\left.\begin{array}{r} x \\ \text{£}1 \\ 1\text{ yr.} \end{array}\right\} = \left\{\begin{array}{l} 325{\cdot}3166 \\ \quad 1{\cdot}03237\text{, amount of £1 in one year.} \\ \quad 7{\cdot}1233\text{ yrs.} \end{array}\right.$$

The above statement would give the amount at *simple* interest, but the question involves a series of equations. If we first find the amount for eight years, then for seven years, by raising the logs of the ratios respectively to the eighth and seventh powers, the difference of these two will give the amount added to the capital in the eighth year, which, counted by $\frac{1233}{10,000}$ths of a year, the decimal, and added to the amount for seven years, will be the required answer. But the shortest method will be to multiply the log of the ratio by the time, and add to the sum the log of the principal, according to the following statement:—

$$\left.\begin{array}{r} x \\ \text{£}1 \\ 1\text{ yr.} \end{array}\right\} = \left\{\begin{array}{l} 325{\cdot}3166 \\ \quad (1{\cdot}03237)^1 \\ \quad 7{\cdot}1233 \end{array}\right\} =$$

$$\begin{array}{l} \log 0{\cdot}0138354 \\ \quad 3321{\cdot}7 \text{ multiplier inverted.} \\ \hline 0{\cdot}0985535 \\ 2{\cdot}5123063 \text{ log of } 325{\cdot}3166 \\ \hline 2{\cdot}6108598 = 408{\cdot}187\text{, or £408 3}s\text{. 9}d. \end{array}$$

What is the amount by the table of £452 6*s.* in nine years, at 4 per cent. per annum, compound interest?

Statement.

What is the amount in nine years of £452·3, if in nine years £1 amount to 1·4233?

$$\left.\begin{array}{r} x \\ 1 \end{array}\right\} = \left\{\begin{array}{l} 452{\cdot}3 \\ \quad 1{\cdot}4233 \end{array}\right\} = 643{\cdot}7585\text{, or £643 15}s\text{ 2}d.$$

Required the amount of £452 6*s*., in nine years, at 4 per cent. compound interest, convertible half-yearly?

$$9 \text{ years} \times 2 = 18 \text{ intervals}$$
$$4 \text{ per cent.} \div 2 = 2 \text{ per cent.}$$

$$\left.\begin{matrix} x \\ 1 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 452{\cdot}3 \\ (1{\cdot}02)^1 \\ 18 \text{ intervals} \end{matrix}\right\} =$$

log 0·0086002
18

0·1548036 = 1·428247, amount of £1 in nine years.

452·3 = log 2·6554266

2·8102302 = 645·9964, or £645 19*s*. 11¼*d*

The Principal.

21°. What sum will amount to £432 in eight years, at 3 per cent compound interest?

Statement.

What principal in eight years will amount to £432, if £1·2668 be produced in eight years by £1?

$$\left.\begin{matrix} x \\ 1{\cdot}2668 \end{matrix}\right\} = \left\{\begin{matrix} 432 \\ 1 \end{matrix}\right\} = 341{\cdot}0167, \text{ or } £341\ 0s.\ 4d.$$

What sum will amount to £432 in eight years, at 3 per cent. compound interest, convertible quarterly?

$$\left(1 + \frac{03}{4}\right)^{8\times4} = (1 + {\cdot}0075)^{32}, \text{ amount of } £1 \text{ at } 32 \text{ intervals,}$$

$$\left.\begin{matrix} x \\ (1{\cdot}0075)^1 \\ 32 \text{ intervals for} \end{matrix}\right\} \left\{\begin{matrix} 432 \\ 1 \\ 1 \end{matrix}\right\} = 340{\cdot}1257, £340\ 2s.\ 6\tfrac{1}{4}d.$$

The *greater* the number of intervals, the *less* the sum required to produce the given amount; the two last terms are therefore inverted.

The Rate per Cent., and Time.

22°. At what rate per cent. compound interest, convertible half-yearly, may a sum of £369, due five years hence, be discharged by the immediate payment of £263 6*s*. 9*d*.?

The difference between the logs of the principal and of the sum due, equals the log of the amount of £1 at the end of the term, which is the amount of one interval raised to the power of the number of intervals. This log, then, measured by the number of intervals, gives the log of the amount of £1 for the first interval, which, counted by the intervals in a year, will give the rate per pound for one year.

$$\begin{array}{rl} 369 & = \log 2{\cdot}5670264 \\ 263{\cdot}3375 & = \log 2{\cdot}4205116 \\ \hline & 10)\ 0{\cdot}1465148 \\ \hline & 0{\cdot}0146514 = \end{array}$$

$$\begin{array}{rl} 1{\cdot}0343 & \\ 1 & \\ \hline {\cdot}0343 & \text{Interest for the half-year.} \\ 2 & \\ \hline {\cdot}0686 & \text{Rate per pound per annum.} \\ \div\ 100 & \\ \hline 6{\cdot}86 & \text{Rate per cent. per annum, or £6 } 17s.\ 2\tfrac{1}{2}d. \end{array}$$

In how many years will £452 6*s*., at 4 per cent. compound interest, convertible half-yearly, amount to £645 19*s*. 11¼*d*.

The amount of £1 at the end of the term equals the sum divided by the principal, or the difference of the logs of the sum and principal equals the log of the amount of £1 in one year, raised to the power of the number of years. This log contains the amount of £1, at the end of the first interval, as many times as the number of intervals. To find the number of years, this divisor must be counted by the number of periods of conversion in one year

$$\begin{array}{rl} & 2{\cdot}8102302 \text{ log of sum.} \\ & 2{\cdot}6554266 \\ \hline \text{Log of } (1{\cdot}021) \times 2 = 0{\cdot}0172004) & 0{\cdot}1548036\ (9 \text{ years.} \\ & 0{\cdot}1548036 \\ \hline \\ \hline \end{array}$$

TABLE IV.

Present Value of £1, due at the End of any Number of Years, not exceeding fifty.

Years.	3 per Cent.	4 per Cent.	5 per Cent.
1	·9708 7397	·9615 3846	·9523 8095
2	·9425 9591	·9245 5621	·9070 2948
3	·9151 4166	·8889 9636	·8638 3670
4	·8884 8705	·8548 0419	·8227 0247
5	·8626 0878	·8219 2711	·7835 2616
6	·8374 8426	·7903 1453	·7462 1540
7	·8130 9151	·7599 1781	·7106 8133
8	·7894 0923	·7306 9020	·6768 3936
9	·7664 1673	·7025 8674	·6446 0892
10	·7440 9391	6755 6417	·6139 1325
11	·7224 2126	·6495 8093	·5846 7929
12	·7013 7988	·6245 9705	·5568 3742
13	·6809 5134	·6005 7409	·5303 2135
14	·6611 1781	·5774 7508	·5050 6795
15	·6418 6195	·5552 9450	·4810 1710
16	·6231 6694	·5339 0818	·4581 1152
17	·6050 1645	·5133 7325	·4362 9669
18	·5873 9461	·4936 2812	·4155 2065
19	·5702 8603	4746 4242	·3957 3396
20	·5536 7575	·4563 8695	·3768 8948
21	·5375 4928	·4388 3360	·3589 4236
22	·5218 9250	·4219 5539	·3418 4987
23	·5066 9175	·4057 2633	·3255 7131
24	·4919 3374	·3901 2147	·3100 6791
25	·4776 0556	·3751 1680	·2953 0277
26	·4636 9473	·3606 8923	·2812 4073
27	·4501 8906	·3468 1657	·2678 4832
28	·4370 7675	·3334 7747	·2550 9364
29	·4243 4636	·3206 5141	·2429 4632
30	·4119 8676	·3083 1867	·2313 7745
31	·3999 8714	·2964 6026	·2203 5947
32	·3883 3703	·2850 5794	·2098 6617
33	·3770 2625	·2740 9417	·1998 7254
34	·3660 4490	·2635 5209	·1903 5480
35	·3553 8340	·2534 1547	·1812 9029
36	·3450 3243	·2436 6872	·1726 5741
37	·3349 8294	·2342 9685	·1644 3563
38	·3252 2615	·2252 8543	·1566 0536
39	·3157 5355	·2166 2061	·1491 4797
40	·3065 5684	·2082 8904	·1420 4568
41	·2976 2800	·2002 7792	·1352 8160
42	·2889 5922	·1925 7493	·1288 3962
43	·2805 4294	·1851 6820	·1227 0440
44	·2723 7178	·1780 4635	·1168 6133
45	·2644 3862	·1711 9841	·1112 9651
46	·2567 3652	·1646 1386	·1059 9668
47	2492 5877	·1582 8256	·1009 4921
48	·2419 9880	·1521 9476	·0961 4211
49	·2349 5029	·1463 4112	·0915 6391
50	·2281 0708	·1407 1262	·0872 0373

PRESENT VALUE OF SUMS AT COMPOUND INTEREST.

23°. The present value of £1 to be received at the end of one year is the sum which, at compound interest, will produce that amount at the end of the term. The amount of £1 at 3 per cent. compound interest, payable at the end of the year, may be thus expressed $(1 + \cdot 03)^1$. The *present value* of this sum, being the reverse of this operation, is expressed algebraically by $(1 + \cdot 03)^{-1}$, which signifies the reciprocal of $1 + \cdot 03$, or $\frac{1}{1 + \cdot 03} = \cdot 9708$. One pound to be received at the end of two years equals $(1 + \cdot 03)^{-2}$, or $\frac{1}{1 + \cdot 03^{-2}}$; £1 at three years equals $(1 + \cdot 03)^{-3}$, &c.

On this principle Table IV. is constructed, which gives the present values of £1 at various rates per cent. from one to fifty years, from which the present value of any other sum at the same rate may be found.

Examples.

What is the present value of £237, due at the end of seventeen years, at 4 per cent. compound interest?

Statement.

What sum in seventeen years will produce £237, if in the same time £1·9479 be produced by £1?

$$\left.\begin{matrix} x \\ 1\cdot 9479 \end{matrix}\right\} = \left\{\begin{matrix} 237 \\ 1 \end{matrix}\right\} = 121\cdot 67$$

By Table IV. the question becomes a *direct* proportion.

What is the present value of £237, due in a given time, if the present value of £1 in the same time equals ·5134?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 237 & \\ & \cdot 5134 \end{matrix}\right\} 121\cdot 67, \text{ or } £121\ 13s.\ 5d.$$

What sum will a present payment of £219 7*s.* 6*d.* entitle a person to at the end of twelve years, interest 3 per cent.?

Statement.

What will be the amount in twelve years, of 219·375 if in the same time £1 produce 1·4258?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 219{\cdot}375 \\ 1{\cdot}4258 \end{matrix}\right\} = 312{\cdot}7848, \textit{ i. e., } £312\ 15s.\ 8\tfrac{1}{2}d.$$

The sum of £249 7*s*. 8*d*. was paid for the present value of a sum to be received eight years hence; what will the person making the payment be then entitled to, allowing 8 per cent. compound interest, payable quarterly?

Statement.

What is the amount of £249·383; if in one interval £1 produce 1·02, how much in 32 intervals?

N.B. The interest of £1 in the first interval is $\frac{1}{4}$ of ·08, or ·02, and there are $8 \times 4 = 32$ intervals.

$$\left.\begin{matrix} x \\ 1 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 249{\cdot}383 \\ (1{\cdot}02)^1 \\ 32 \end{matrix}\right. \left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} \log 0{\cdot}0086002 \\ 32 \end{matrix}$$

$$\begin{array}{l} \hline 0{\cdot}2752064 \\ 2{\cdot}3968673 \ \log \text{ of } 249{\cdot}383. \\ \hline 2{\cdot}6720737 = 469{\cdot}973, \textit{ i.e. } £469\ 19s.\ 5\tfrac{3}{4}d. \end{array}$$

What is the present value of £350, due seven years hence, allowing 6 per cent. compound interest, payable quarterly?

Statement.

Required the present value of £350, interest convertible in 28 intervals, if $(1{\cdot}015)^1$ be produced in one interval by £1

$$\left.\begin{matrix} x \\ (1{\cdot}015)^1 \times 28 \end{matrix}\right\} = \left\{\begin{matrix} 350 \\ 1 \end{matrix}\right\} = 230{\cdot}785, \text{ or } £230\ 15s.\ 8\tfrac{1}{2}d.,$$

or log 0·1810480.

The Time.

24°. A legacy of £1500 was exchanged for a present pay-

ment of £1219 12*s.* 9*d.*, reckoning interest at 3 per cent. How many years hence was the legacy due?

The difference of the logs of the present value and the sum due equals the log of the amount of £1, at the end of the term, which must be measured by the log of the amount of £1 at the end of one interval to give the number of years or intervals.

£1500 = log 3·1760913
1219·637 = log 3·0862305

1·03 = log 0·0128372) 0·0898608(7 years
0·0898608

Or, by Table IV., what is the present value of £1, if 1219·637 be the present value of £1500?

$$\left.\begin{matrix} x \\ 1500 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 1219{\cdot}637 \end{matrix}\right\} = {\cdot}81309, \text{ present value of £1 in 7 yrs.}$$

The sum of £249 7*s.* 8*d.* was paid down in lieu of £469 19*s.* 6*d.*, the discount allowed being 8 per cent. compound interest, payable quarterly. How long was the sum paid before due?

Amount in one interval,

1·02 = log 0·0086002 2·6720737 log of 469·973.
4 2·3968673 log of 249·383.

Amount in one year 0·0344008) 0·2752064 (8 years.
0·2752064

The Rate per Cent.

25°. A owes B £327, payable at the expiration of twelve years, which A is allowed to discharge by the immediate payment of £204 4*s.* 10½*d.*; what is the rate per cent. compound interest?

$$\left.\begin{matrix} x \\ 327 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 204{\cdot}243 \end{matrix}\right\} = {\cdot}62459 \left\{\begin{matrix} \text{Present value by Table IV.} \\ \text{of £1 in twelve years,} \\ \text{at 4 per cent} \end{matrix}\right.$$

Amount of £1 per *Annum* in any Number of Years, not exceeding fifty.

Years.	3 per Cent.	4 per Cent.	5 per Cent.
1	1·000000	1·000000	1·000000
2	2·030000	2·040000	2·050000
3	3·090900	3·121600	3·152500
4	4·183627	4·246464	4·310125
5	5·309136	5·416323	5·525631
6	6·468410	6·632975	6·801913
7	7·662462	7·898294	8·142008
8	8·892336	9·214226	9·549109
9	10·159106	10·582795	11·026564
10	11·463879	12·006107	12·577893
11	12·807796	13·486351	14·206787
12	14·192030	15·025805	15·917127
13	15·617790	16·626838	17·712983
14	17·086324	18·291911	19·598632
15	18·598914	20·023588	21·578564
16	20·156881	21·824531	23·657492
17	21·761588	23·697512	25·840366
18	23·414435	25·645413	28·132385
19	25·116868	27·671229	30·539004
20	26·870374	29·778079	33·065954
21	28·676486	31·969202	35·719252
22	30·536780	34·247970	38·505214
23	32·452884	36·617889	41·430475
24	34·426470	39·082604	44·501999
25	36·459264	41·645908	47·727099
26	38·553042	44·311745	51·113454
27	40·709634	47·084214	54·669126
28	42·930923	49·967583	58·402583
29	45·218850	52·966286	62·322712
30	47·575416	56·084938	66·438848
31	50·002678	59·328335	70·760790
32	52·502759	62·701469	75·298829
33	55·077841	66·209527	80·063771
34	57·730177	69·857909	85·066959
35	60·462082	73·652225	90·320307
36	63·275944	77·598314	95·836323
37	66·174223	80·702246	101·628139
38	69·159449	85·970336	107·709546
39	72·234233	90·409150	114·095023
40	75·401260	95·025516	120·799774
41	78·663298	99·826536	127·839763
42	82·023196	104·819598	135·231751
43	85·483892	110·012382	142·993339
44	89·048409	115·412877	151·143006
45	92·719861	121·029392	159·700156
46	96·501457	126·870568	168·685164
47	100·396501	132·945390	178·119422
48	104·408396	139·263206	188·025393
49	108·540648	145·833734	198·426663
50	112·796867	152·667084	209·347996

Or, the difference of the logs of the sum due and present value gives the log of the amount of £1 in twelve years, at the given rate, which, divided by twelve, gives the amount, with interest, of £1 at the end of one interval:

$$\begin{array}{rl} 327 = \log & 2{\cdot}5145478 \\ 204{\cdot}243 = \log & 2{\cdot}3101471 \\ \hline & 12)0{\cdot}2044007 \\ \hline & 0{\cdot}0170333 = 1{\cdot}04, \text{ or 4 per cent.} \end{array}$$

At what rate per cent. compound interest, payable quarterly, may a sum of £469 19*s.* 6*d.*, due in eight years, be discharged by the immediate payment of £249 7*s.* 8*d.*?

$$\begin{array}{l} \text{Log of sum} = 2{\cdot}6720737 \\ \text{Log of principal} = 2{\cdot}3968673 \\ \hline 0{\cdot}2752064 \div 32 \text{ intervals} = {\cdot}00860645 \\ = 1{\cdot}02 \text{ per quarter.} \\ \quad 4 \\ \hline {\cdot}08 \text{ per pound per annum, or 8 per cent.} \end{array}$$

ANNUITIES AT COMPOUND INTEREST.

26°. The sum of a series, in which each term is multiplied by a common difference, equals the first term, multiplied by the difference between unity and that power of the common ratio whose index is equal to the number of terms, divided by the product of the difference between unity and the common ratio. A series of this kind is called a "geometrical progression," and the sum of its terms, taken at each step from 1 to 50, is the principle on which Table V. is constructed, which gives the amount of £1 *per annum* at various rates per cent., the common ratio being the interest, thus:

What is the amount of £1 per annum, at 4 per cent. compound interest, in five years?

0 year.	1 year.	2 years.	3 years.	4 years.	5 years.
£1	$(1+\cdot04)$	$(1+\cdot04)^2$	$(1+\cdot04)^3$	$(1+\cdot04)^4$	$(1+\cdot04)^5$.
Amount of 1st payment.	of 2nd do.	of 3rd do.	of 4th do.	of 5th do.	of 6th do.

Statement.

What is the amount in five years of £1 per annum if ·04 is its interest for one year? But instead of one year we have five, minus one year's interest.

$$\left.\begin{matrix} x \\ \cdot04 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 1 \\ (1+\cdot04)^5 - 1 \end{matrix}\right\} = \frac{(1+\cdot04)^5 - 1}{\cdot04}, \text{ or } £5\cdot4163.$$

The amount of an annuity is the sum of a series of terms in geometrical progression. On inspecting the above series, it appears that the first payment is simply £1, and that the series contains six times the annuity and five times the interest; and that the amount in one year is multiplied by itself four times, or the given number of years minus one. The nth power of a number, we have seen, is that number multiplied by itself $n-1$ times, therefore a number multiplied by itself n times, is the $n+1$th power, The rule, then, is—raise the amount of £1 in one year to the power of the number of years, deduct unity from the product, and divide by the interest of £1 for one year.

Examples.

The Amount.

What will an annuity of £33 17*s.* 9*d.* amount to in fourteen years, at 4 per cent. compound interest?

Statement.

What is the amount of £33·8875 per annum, if ·04 be the interest of £1 for one year, and fourteen years' interest of £1 equals ·7317?

$(1+\cdot04)^{14} = 1\cdot7317$, amount of £1 in fourteen years.

$$\left.\begin{matrix} x \\ \cdot04 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 33\cdot8875 \\ 1 \\ \cdot7317 \end{matrix}\right\} = 619\cdot866.$$

By Table V.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 33\cdot8875 \\ 18\cdot2919 \end{matrix}\right\} = 619\cdot866, \text{ i.e., £619 17s. 4d.}$$

What will be the amount of an annuity of £234 in eight years, at £4 6s. per cent. compound interest?

$$(1 + \cdot043)^8 = 1\cdot400471. \quad \text{Interest} = \cdot400471.$$

$$\left.\begin{matrix} x \\ \cdot043 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 234 & & \\ & 1 & \\ & & \cdot40047 \end{matrix}\right\} = 2179\cdot342, \text{ i.e., £2179 6s. } 10\tfrac{1}{4}d.$$

The Annuity.

27°. The usual rule for finding the annuity is—"Multiply the amount of the annuity by the interest of £1 for one year, and divide the product by the amount of £1 in the given time, less one."

The interest of £1 for one year must be counted by the number of pounds or parts of pounds in the sum, the sum being composed of one year's amount, multiplied by itself $n - 1$ times, which will give $n - 1$ times the compound interest of £1; this, measured by the amount of £1 in n years, less one, will give the $n - 1$th part of one year's income or annuity.

What annuity, accumulating at $3\frac{3}{4}$ per cent. compound interest, for forty years, will amount to £600?

Statement.

What annuity will amount to £600, if the interest of £1, in forty years, equals 3·3604, and the interest per pound = ·0375?

This is an inverse proportion; the greater the annuity the fewer times will it be contained in 600 times, the interest.

By logs, the 40th power of 1·0375 = 4·36037
Deduct principal 1

3·36037

$$\left.\begin{matrix} x \\ 3\cdot3604 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 600 & & \\ & 1 & \\ & & \cdot0375 \end{matrix}\right\} = 6\cdot6956, \text{ i.e., £6 13s. 11d}$$

At the expiration of ten years, £289 will be required for the renewal of a lease; what sum, at 5 per cent. compound interest, should be annually laid by to produce that amount?

By Table V. the amount of £1 per annum, in ten years, equals 12·5779.

$$\left.\begin{matrix} x \\ 12{\cdot}5779 \end{matrix}\right\} = \left\{\begin{matrix} 289 \\ 1 \end{matrix}\right\} = 22{\cdot}9768, \textit{ i. e.}, \text{£}22\ 19s.\ 6\tfrac{1}{2}d.$$

The Time.

28°. Find the interest of £1 for the number of years, which, divided by the annuity and unity added, will give the amount of £1, at the given rate, for the whole term. The log of this product contains the log of the amount of £1 in one year as many times as the number of years in the term.

In how many years will an annuity of £8 per annum amount to £187 6s. $3\frac{3}{4}d.$, at 3 per cent. compound interest?

$$\left.\begin{matrix} x \\ 8 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 187{\cdot}3155 \\ 1 \\ {\cdot}03 \end{matrix}\right\} \begin{matrix} = {\cdot}70243 + 1 = 1{\cdot}70243 \\ = \log \left.\dfrac{0{\cdot}2310734}{0{\cdot}0128372}\right\} = 18 \text{ years.} \end{matrix}$$

29°. *The Rate per Cent.*

is so involved in questions of this kind, that no concise method can be given by which it may be accurately brought out; nor is there any algebraic process yet discovered that affords any other than an approximate solution.

At what rate per cent. will £25 per annum amount in ten years to £347 12s. $8\frac{1}{4}d.$?

The following statement will give the amount of £1 per annum for the whole term:

$$\left.\begin{matrix} x \\ 25 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 347{\cdot}634 \end{matrix}\right\} = 13{\cdot}90536.$$

On comparing this result with a table, it will be seen that the amount of £1 per annum, at 7 per cent., in ten years, equals 13·81644, which, deducted from 13·90536, leaves a difference of ·08892, or about 1s. $9\frac{1}{4}d.$ more than 7 per cent

TABLE VI.

Present value of £1 per Annum for any Number of Years, not exceeding fift

Years.	3 per Cent.	4 per Cent.	5 per Cent.
1	·970874	·961538	·952381
2	1·913470	1·886095	1·859410
3	2·828611	2·775091	2·723248
4	3·717098	3·629895	3·545951
5	4·579707	4·451822	4·329477
6	5·417191	5·242137	5·075692
7	6·230283	6·002055	5·786373
8	7·019692	6·732745	6·463213
9	7·786109	7·435332	7·107822
10	8·530203	8 110896	7·721735
11	9·252624	8·760477	8·306414
12	9·954004	9·385074	8·863252
13	10·634955	9·985648	9·393573
14	11·296073	10·563123	9·898641
15	11·937935	11·118387	10·379658
16	12·561102	11·562296	10·837770
17	13·166118	12·165669	11·274066
18	13·753513	12·659297	11·689587
19	14·323799	13·133939	12·085321
20	14·877475	13·590326	12·462210
21	15·415024	14·029160	12·821153
22	15·936917	14·451115	13·163003
23	16·443608	14·856842	13·488574
24	16·935542	15·246963	13·798642
25	17·413148	15·622080	14·093945
26	17·876842	15·982769	14·375185
27	18·327031	16·329586	14·643034
28	18·764108	16·663063	14·898127
29	19·188455	16·983715	15·141074
30	19·600441	17·292033	15·372451
31	20·000428	17·588494	15·592811
32	20·388766	17·873552	15·802677
33	20·765792	18·147646	16·002549
34	21·131837	18·411198	16·192904
35	21·487220	18·664613	16·374194
36	21·832252	18·908282	16·546852
37	22·167235	19·142579	16·711287
38	22·492462	19·367864	16·867893
39	22·808215	19·584485	17·017041
40	23·114772	19·792774	17·159086
41	23·412400	19·993052	17·294368
42	23·701359	20·185627	17·423208
43	23·981902	20·370795	17·545912
44	24·254274	20·548841	17·662773
45	24·518713	20·720040	17·774070
46	24·775449	20·884654	17·880067
47	25·024708	21·042936	17·981016
48	25·266707	21·195131	18·077158
49	25·501657	21·341472	18·168722
50	25·729764	21·482185	18·255925

What will an annuity of £25 amount to in nine years, at 6 per cent. compound interest, when annuity and interest are payable half-yearly?

$$\left.\begin{matrix} x \\ 1 \\ \cdot 06 \end{matrix}\right\} = \left\{\begin{matrix} 25 \\ (1\cdot 03)^{18}, \text{ or } \cdot 7024 \text{ int.} \\ 1 \end{matrix}\right\} = 292\cdot 67, \textit{i.e.}, £292\ 13s.\ 6d.$$

Or by Table V.

What is the amount of half an annuity of £25, if £1 in eighteen years, at 3 per cent., equals 23·414?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 12\frac{1}{2} \\ 23\cdot 414 \end{matrix}\right\} = £292\cdot 674.$$

PRESENT VALUES OF ANNUITIES AT COMPOUND INTEREST.

30°. If there were no interest of money, the amount of £1 to be received at the end of one year would be the same as £1 received now, *i. e.*, $\frac{1}{1} = 1$; but if ·03 interest can be made in a year of £1, the fractions become

$$\frac{1}{1 + \cdot 03} \quad \frac{1}{(1 + \cdot 03)^2} \quad \frac{1}{(1 + \cdot 03)^3} \quad \frac{1}{(1 + \cdot 03)^4}$$

being, respectively, the present values of £1, to be received at the end of 1, 2, 3, and 4 years.

If we deduct from £1 the whole amount of the sum that would produce it, at the end of four years, what remains is £1, minus its present value if paid four years hence, or the discount of £1; for, as each pound is to be paid at the end of each year, there is the present values of £1, for 1, 2, 3, &c., years, to be deducted respectively from each pound. We make the whole deduction from £1, and the remainder will contain the interest of £1 for one year, as many times as the years by which it has been reduced.

Examples.

What is the present value of £1 per annum for four years, if 1 — ·8884 is the discount for four years, and ·03 is produced in one year by £1?

$$\left.\begin{array}{r} x \\ 1 \\ \cdot 03 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ 1 - \cdot 8884 \\ 1 \end{array}\right\} = 3\cdot 717$$

What is the present value, at 5 per cent., of the lease of an estate for nineteen years, worth £214 per annum, the rent being £60 per annum?

Deduct the rent from the income for the value of the annuity which is to be purchased; 214 — 60 = £154. Present value by Table VI., of £1 per annum for nineteen years, = 12·0853

$$\left.\begin{array}{r} x \\ 1 \end{array}\right\} = \left\{\begin{array}{l} 154 \\ \quad 12\cdot 0853 \end{array}\right\} = 1861\cdot 13 = £1861\ 2s.\ 7\tfrac{1}{4}d.$$

What sum will be required for the purchase of an annuity of £25, to continue twelve years, interest £4 5s. per cent.?

Log of 1 = 0·0000000
0·0180761 log of 1·0425, amount of £1 in one year

$\bar{1}\cdot 9819239$ log of present value of £1, to be received
12 in one year.

$\bar{1}\cdot 7830868$ = ·60685, present value of £1 in twelve years.

Statement.

What is the value of an annuity of £25, if present value of £1 in twelve years equals (1 — ·60685), and ·0425 is the interest of £1 for one year?

$$\left.\begin{array}{r} x \\ 1 \\ \cdot 0425 \end{array}\right\} = \left\{\begin{array}{l} 25 \\ (1 - \cdot 60685),\text{ or } \cdot 39314 \\ 1 \end{array}\right\} = 231\cdot 2612,$$

or £231 5s. $2\frac{1}{2}d.$

For how many years may an annuity of £25 per annum, be purchased for £231 5s. $2\frac{1}{2}d.$, interest £4 5s. per cent.?

The 25th part of the present value will be the present value of £1 per annum, for the whole term. By comparing this amount with Table VI., the column nearest to the amount of interest will give the number of years.

$$\left.\begin{matrix} x \\ 25 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 231\cdot2612 \end{matrix}\right\} = 9\cdot25045.$$

which reads—what is the present value of £1 per annum, in the number of years, if £25 per annum equal 231·2612. The answer 9·25045, comes the nearest to the tabular value at twelve years, under the column headed 4 per cent.

An annuity of £25 per annum for 23 years, was sold for £800 19*s.*; what was the rate per cent.?

$$\left.\begin{matrix} x \\ 25 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 800\cdot95 \end{matrix}\right\} = 12\cdot307$$

Tabular value of £1 per annum, for 23 years, at nearly 6 per cent.

PERPETUITIES.

31° An annuity to continue for ever is called a perpetuity. The present value of a perpetuity is the sum that will produce the annual interest at the given rate per cent.

Examples.

What is the present value, at 3 per cent., of a perpetuity of £3 per annum?

$$\left.\begin{matrix} x \\ \cdot03 \end{matrix}\right\} = \left\{\begin{matrix} 3 \\ 1 \end{matrix}\right\} = £99.$$

What is the present value of an estate in fee simple, of £563, when the interest of money is 5 per cent.?

$$\left.\begin{matrix} x \\ \cdot05 \end{matrix}\right\} = \left\{\begin{matrix} 563 \\ 1 \end{matrix}\right\} = £11,260.$$

What perpetuity will £856 purchase, when the rate of interest is $3\frac{1}{2}$ per cent.?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 856 & \\ & \cdot035 \end{matrix}\right\} 29\cdot96, \textit{ i. e. } £29\ 19s.\ 2\tfrac{1}{2}d$$

REVERSIONS

32°. An annuity which is not to be entered upon until after the expiration of a term of years, is called a reversionary or deferred annuity.

The present value of a deferred annuity is the difference between the present value of an annuity to begin immediately and continue until the expiration of the reversion, and an annuity to continue until the time of entering on the reversion.

What is the present value of the reversion of £40 per annum, for seven years, to be entered upon after the expiration of twelve years; interest 4 per cent.?

13·13393	present value of £1	per annum	for 19 years.
9·38507	,,	,,	for 12 years.
3·74886	,,	,,	for 7 years, deferred 12 years.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 40 \\ 3{\cdot}74886 \end{matrix}\right\} = 149{\cdot}9544, \text{ or } £149\ 19s.\ 1d.$$

What Deferred Annuity may be purchased for a given sum?

The difference between the present values of £1 per annum for the whole term, and the present value of £1 per annum to continue until the commencement of the deferred term, will be the present value of £1 annuity for the deferred term. Divide the purchase money by this difference for the annuity required.

What annuity, to continue nine years after the expiration of the next twelve years, may be purchased for £140; interest 5 per cent.?

12·821153	present value of £1	per annum	for 21 years
8·863252	,,	,,	for 12 years.
3·957901	,,	,,	for the deferred term.

$$\left.\begin{matrix} x \\ 3{\cdot}9579 \end{matrix}\right\} = \left\{\begin{matrix} 140 \\ 1 \end{matrix}\right\} = 35{\cdot}3723, \textit{ i. e}, £35\ 7s.\ 5\tfrac{1}{2}d.$$

REVERSIONS IN PERPETUITY.

33°. Required the present value of the reversion of the perpetuity of £62 per annum, to be received at the expiration of fourteen years, at 4 per cent.

Present value, by table, of £1 due in 14 years = ·5051.

$$\left.\begin{matrix} x \\ 1 \\ \cdot 04 \end{matrix}\right\} = \left\{\begin{matrix} 62 & \\ & \cdot 5051 \\ 1 & \end{matrix}\right\} = £782\ 18s.\ 8d.$$

The reversion of a fee simple estate, after twelve years, is sold for £640 17*s.* 5*d.*; what annual return should it produce to allow the purchaser 4 per cent. interest for his money?

The purchase money, counted by the interest of £1, gives the whole annual income for the present time; but as this income is to be deferred twelve years, the present income must be counted by the amount of £1 deferred to that period.

$$\left.\begin{matrix} x \\ 1 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 640\cdot 8708 \\ \cdot 04 \\ 1\cdot 601 \end{matrix}\right\} = 41\cdot 0413,\ i.\ e.,\ £41\ 0s.\ 10d.$$

RENEWAL OF LEASES

34°. The present value of an annuity deferred for the unexpired term of the lease, and then to continue for the period renewed, will be the fine required for the renewal of any number of years expired in a lease.

Fifty years having expired in a lease for the term of sixty years, what sum should be paid for renewing them, supposing the estate to produce a clear rental of £240 per annum, and the interest of money 5 per cent.?

18·9293 present value of £1 per annum for 60 years.
7·7217 „ „ for 10 years, the unexpired term.

11·2076 present value of an annuity of £1 for 50 years.
240

2689·824 = £2689 16*s.* 5¾*d.*

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 240 & \\ & 11\cdot 2076 \end{matrix}\right\} = £2689\ 16s.\ 5\tfrac{3}{4}d.$$

Thirty years having expired in a lease for forty years, what fine will be required for renewing ten years of the same, supposing the yearly rental £70, and the rate of interest 4 per cent.?

13·5903 present value of £1 per annum for 20 years.
8·1109 ,, ,, for 10 years, the unexpired term.

5·3794 present value of an annuity of £1 for 10 years.
70

383·558

$$\left.\begin{array}{l} x \\ 1 \end{array}\right\} = \left\{\begin{array}{ll} 70 & \\ & 5{\cdot}4794 \end{array}\right\} = £383\ 11s.\ 1\tfrac{3}{4}d.$$

MISCELLANEOUS CALCULATIONS.

35°. What is the interest of £462 10*s*., for six years and five days, at 4 per cent. ?

$$\left.\begin{array}{l} x \\ 1 \\ 1 \end{array}\right\} = \left\{\begin{array}{r} 462{\cdot}5 \\ {\cdot}04 \\ 6{\cdot}0136 \end{array}\right\} = £111\ 5s.\ 0\tfrac{1}{4}d.$$

What will be the amount, in seven years, of £756, at $4\frac{1}{2}$ per cent. ?

$$\left.\begin{array}{l} x \\ 1 \\ 1 \end{array}\right\} = \left\{\begin{array}{lr} 756 & \\ & {\cdot}045 \\ 7 & \end{array}\right\} = \overset{\text{Interest.}}{238{\cdot}14} + \overset{\text{Principal.}}{756} = £994\ 2s.\ 9\tfrac{3}{4}d.$$

Required the interest of £7000 for 230 days, at 4 per cent.

$$\left.\begin{array}{l} x \\ 1 \\ 1 \end{array}\right\} = \left\{\begin{array}{lr} 7000 & \\ & {\cdot}04 \\ & {\cdot}6301 \end{array}\right\} = 176{\cdot}428,\ i.e.,\ £176\ 8s\ 6\tfrac{1}{2}d.$$

What is the amount of £573, in six years, at 5 per cent. compound interest?

Amount by Table III. of £1 in six years, 1·3401.

$$\left.\begin{array}{l} x \\ 1 \end{array}\right\} = \left\{\begin{array}{lr} 573 & \\ & 1{\cdot}3401 \end{array}\right\} = £767\ 17s.\ 6\tfrac{1}{2}d.$$

Required the amount of £346 17*s*. 9*d*., in eight years, at 4 per cent. compound interest.

$$\left.\begin{array}{l} x \\ 1 \end{array}\right\} = \left\{\begin{array}{r} 346{\cdot}8875 \\ 1{\cdot}3686 \end{array}\right\} = £474\ 15s.$$

What is the present value of £943, to be received at the end of fifteen years, reckoning 3 per cent. compound interest?

Present value by Table IV. of £1 due in fifteen years, 6419.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 943 & \\ & \cdot6419 \end{matrix}\right\} = £605\ 6s.\ 5\tfrac{1}{4}d.$$

Required the amount, by Table V., of £60 per annum in nine years, at 4 per cent. compound interest.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 60 \\ 10\cdot5828 \end{matrix}\right\} = £634\ 19s.\ 4\tfrac{1}{2}d.$$

What is the present value, by Table VI., of a temporary annuity of £75 18s., for twelve years, at 4 per cent. compound interest?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 75\cdot9 \\ 9\cdot385 \end{matrix}\right\} = £712\ 6s.\ 6\tfrac{1}{4}d.$$

What annuity, by Table VI., may be purchased for £639 17s. 4d., for nine years, at 4 per cent. compound interest?

$$\left.\begin{matrix} x \\ 7\cdot4353 \end{matrix}\right\} = \left\{\begin{matrix} 639\cdot8667 \\ 1 \end{matrix}\right\} = £86\ 1s.\ 1\tfrac{1}{2}d.$$

Required the present value of a deferred annuity of £80, to be entered upon at the expiration of twelve years, and then to be continued for seven years, at 4 per cent. compound interest.

13·1339 pres. val., by Table VI., of £1 per an. for 19 years.
9·3851 ,, ,, for 12 years.

3·7488 pres. val. of an annuity deferred 12 years to continue 7 years.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 80 \\ 3\cdot7488 \end{matrix}\right\} = £299\ 18s.\ 1d.$$

A freehold estate produces £120 per annum; what is the

present value of the perpetuity, reckoning 4 per cent. interest?

$$\left.\begin{matrix} x \\ 4 \end{matrix}\right\} = \left\{\begin{matrix} 120 \\ 100 \end{matrix}\right\} = £3000, \text{ or } \left.\begin{matrix} x \\ \cdot 04 \end{matrix}\right\} = \left\{\begin{matrix} 120 \\ 1 \end{matrix}\right\} = £3000.$$

A holds a freehold estate of £250 per annum, on a lease, which has twelve years to run; what sum ought B to give, to come into possession of the estate at the end of that time, reckoning interest at 5 per cent.?

$$\left.\begin{matrix} x \\ 1 \\ 5 \end{matrix}\right\} = \left\{\begin{matrix} 250 \\ \cdot 6246 \\ 100 \end{matrix}\right\} = £3123.$$

What perpetuity, to be entered upon fourteen years hence, may be purchased for the sum of £2644 9*s.* 6*d.*, reckoning interest at 3 per cent.?

$$\left.\begin{matrix} x \\ 1 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 2644 \cdot 475 \\ \cdot 03 \\ 1 \cdot 5126 \end{matrix}\right\} = £120.$$

A freehold estate, producing £75 per annum, is mortgaged for the period of fourteen years; what is its present value at 4 per cent.?

$$\left.\begin{matrix} x \\ \cdot 04 \end{matrix}\right\} = \left\{\begin{matrix} 75 \\ 1 \end{matrix}\right\} = £1875, \text{ present value of } £75 \text{ per annum.}$$

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 1875 \\ \cdot 5775 \text{ pres. val. of } £1 \text{ in 14 years.} \end{matrix}\right\} = £1082\ 18s.\ 3d.$$

At how many years' purchase must a freehold estate be bought, that the purchaser may have 5 per cent. for his money; that is, in how many years will the estate produce the money paid as its present value?

$$\left.\begin{matrix} x \\ 5 \end{matrix}\right\} = \left\{\begin{matrix} 100 \\ 1 \text{ yr.} \end{matrix}\right\} = 20 \text{ years.}$$

TABLE VII.

Number of Persons Living and Dying at every Age, according to the Northampton Estimate.

Age.	Living.	Dying.	Age.	Living.	Dying.
0	11650	3000	49	2936	79
1	8650	1367	50	2857	81
2	7283	502	51	2776	82
3	6781	335	52	2694	82
4	6446	197	53	2612	82
5	6249	184	54	2530	82
6	6065	140	55	2448	82
7	5925	110	56	2366	82
8	5815	80	57	2284	82
9	5735	60	58	2202	82
10	5675	52	59	2120	82
11	5623	50	60	2038	82
12	5573	50	61	1956	82
13	5523	50	62	1874	81
14	5473	50	63	1793	81
15	5423	50	64	1712	80
16	5373	53	65	1632	80
17	5320	58	66	1552	80
18	5262	63	67	1472	80
19	5199	67	68	1392	80
20	5132	72	69	1312	80
21	5060	75	70	1232	80
22	4985	75	71	1152	80
23	4910	75	72	1072	80
24	4835	75	73	992	80
25	4760	75	74	912	80
26	4685	75	75	832	80
27	4610	75	76	752	77
28	4535	75	77	675	73
29	4460	75	78	602	68
30	4385	75	79	534	65
31	4310	75	80	469	63
32	4235	75	81	406	60
33	4160	75	82	346	57
34	4085	75	83	289	55
35	4010	75	84	234	48
36	3935	75	85	186	41
37	3860	75	86	145	34
38	3785	75	87	111	28
39	3710	75	88	83	21
40	3635	76	89	62	16
41	3559	77	90	46	12
42	3482	78	91	34	10
43	3404	78	92	24	8
44	3326	78	93	16	7
45	3248	78	94	9	5
46	3170	78	95	4	3
47	3092	78	96	1	1
48	3014	78	97	0	0

PART II.

THEORY OF PROBABILITIES.

36°. Nothing is more uncertain than the duration of life as regards an individual; but when our inquiries involve a large number—for instance, ten thousand persons born alive on the same day—by observing how many survive at the termination of each year, till all are deceased, an average duration of life for every individual may be obtained, sufficiently accurate for all calculations of annuities and life assurance.

The probability of any event happening, is the ratio of the number of favorable cases, or those which contribute to its production, to the number of cases favorable and unfavorable. It appears, by the Carlisle Table of Mortality, that, out of 10,000 persons born alive on one day, 8461 survive the first year. The probability of a child surviving the first year is, then, the ratio 8461 : 10,000, or the fraction $\frac{8461}{10,000}$. The following is the general formula or ratio for the probability of attaining a given age: $\frac{l_{m+n}}{l_m}$; in which l_{m+n} equals the number living at the increased age, and l_m the number living at the present age. If the probability of death before a given age be required, the fraction becomes—

$$\frac{\text{Number dying between the present and increased ages}}{\text{Number living at the present age}}.$$

The theory is applicable to nearly all contingencies, enabling us to calculate the amount of *chance*, or inability to perceive causes, which interferes with the certainty of any

expected event; its investigation is as useful in exposing the fraud or folly of gambling, as in providing against reverses over which we have no control.

Since *probability* is expressed by a fraction, *unity* expresses *certainty*. If a bag contain two white balls, the certainty of drawing a white ball is $\frac{1}{1}$, or unity; but if it contain one white and two black balls, the probability of drawing a white ball becomes the fraction $\frac{1}{3}$, or the ratio 1 to 3, respecting which we should say, that the odds are two to one against drawing a white ball. Let P represent the probability of any event; m, the number of cases favorable to its production; and n, the number of unfavorable cases; then $P = \frac{m}{m+n}$.

The above may afford a general idea of the theory of probabilities sufficient for the practical purposes of this work Those who require a more intimate acquaintance with this interesting branch of mathematical reasoning, may consult the treatise of De Morgan, or that by Jones in his elaborate work on Annuities and Reversions.

We now proceed to the consideration of annuities contingent on individual life, in which this new element will be introduced into our calculations.

Illustrations.

37°. What is the probability, according to the Northampton Table, that a person now aged twenty will attain the age of fifty?

Statement.

What is the individual probability out of 2857 persons living at fifty years of age, on 5132 living at twenty?

$$\left.\begin{matrix} x \\ 5132 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 2857 \end{matrix}\right\} = \cdot 5567, \text{ probability.}$$

What is the probability of a life of forty-five failing to survive one year?

Statement.

What is the individual probability out of 78 persons dying between the ages of forty-five and forty-six, on 3248 persons living at forty-five?

$$\left.\begin{matrix} x \\ 3248 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 78 \end{matrix}\right\} = \cdot024.$$

As unity expresses certainty, the probability of failure is equal to the difference between the probability of attaining a given age and unity.

What is the expectation of life for a child aged three years?

Statement.

What is the individual expectation of life out of 264,834, being the sum of the living at the several ages beyond three years, if 6781 be the number living at three years of age?

$$\left.\begin{matrix} x \\ 6781 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 264834 \end{matrix}\right\} = 39\cdot05 + \cdot5 = 39\cdot55 \text{ years.}$$

Half unity, or ·5, is added to complete the average for those who die in the course of the year.

CONTINGENT OR LIFE ANNUITIES.

38°. In the Northampton Table of Mortality, Table VII., of 11,650 persons born, 8650 survive one year, 7283 survive two years, 6781 survive three years, and so on till they all become extinct.

If, when the interest of money is 3 per cent., it were required to provide at the time of birth £1 for each of the 11,650 who survive one year, it appears that £8650 would be paid among them at the end of a year; the present value of which, $8650 \times (1\cdot03)^{-1}$, is the sum which will provide for the payment of £1 to each survivor, which, divided by 11,650, gives $\dfrac{8650 \times (1\cdot03)^{-1}}{11,650}$, the sum to be contributed on behalf of each.

TABLE VIII.

Expectation of Life at every Age, according to the Northampton Estimate.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
1	32·74	33	26·72	65	10·88
2	37·79	34	26·20	66	10·42
3	39·55	35	25·68	67	9·95
4	40·58	36	25·16	68	9·50
5	40·84	37	24·64	69	9·05
6	41·07	38	24·12	70	8·60
7	41·03	39	23·60	71	8·17
8	40·79	40	23·07	72	7·74
9	40·36	41	22·56	73	7·32
10	39·78	42	22·04	74	6·92
11	39·14	43	21·54	75	6·54
12	38·49	44	21·03	76	6·18
13	37·83	45	20·52	77	5·83
14	37·17	46	20·02	78	5·48
15	36·51	47	19·51	79	5·11
16	35·85	48	19·00	80	4·75
17	35·20	49	18·49	81	4·41
18	34·58	50	17·99	82	4·09
19	33·99	51	17·50	83	3·80
20	33·43	52	17·02	84	3·58
21	32·90	53	16·54	85	3·37
22	32·39	54	16·06	86	3·18
23	31·87	55	15·58	87	3·01
24	31·36	56	15·10	88	2·86
25	30·85	57	14·63	89	2·66
26	30·33	58	14·15	90	2·41
27	29·82	59	13·68	91	2·08
28	29·30	60	13·21	92	1·75
29	28·79	61	12·74	93	1·37
30	28·27	62	12·28	94	1·05
31	27·75	63	11·81	95	·75
32	27·24	64	11·35	96	·50

We have seen that $(1·03)^{-2}$, or $\frac{1}{(1·03)^2}$, is the present value of £1 at 3 per cent., to be certainly received at the end of two years. In the former expression the present value is somewhat less, being diminished by the fraction $\frac{7283}{11650}$, the chance of the individual not surviving the term which would entitle him to the sum.

The present value of the sum to be received at the end of the first year being $(1{\cdot}03)^{-1}$, at the end of the second year $(1{\cdot}03)^{-2}$, the third year $(1{\cdot}03)^{-3}$, &c., multiplied respectively by the probabilities of not surviving that term, the following is the method by which Table IX. is constructed, which gives the present values of an immediate annuity of £1 at the various ages of life.

Multiply the number living at each year of age, by the present value of £1, due at the end of the same number of years as the age; then the present value of the annuity at any age is found by dividing the sum of the products at all the ages *above* that on which the annuity depends, by the product at that age.

The *policy*, or title to an annuity, is received when the purchase-money is paid down; but the first payment of an *immediate* annuity commences at the expiration of the first equal interval at which the annuity is made payable, from the time of entering on possession.

Illustration

What is the present value of an annuity of £1 on a life aged 95, at 3 per cent.?

$$\begin{array}{rl} \cdot 0603 & \text{present value of £1 due in 95 years.} \\ 4 & \text{number living at 95.} \\ \hline \cdot 2412 & \end{array}$$

$$\begin{array}{rl} \cdot 0585 & \text{present value of £1 due in 96 years.} \\ 1 & \text{number living at 96.} \end{array}$$

$$\frac{\cdot 0585 \times 1}{\cdot 0603 \times 4} = \frac{\cdot 0585}{\cdot 2412} = \cdot 242, \text{ pres. val. of annuity of £1 at 95.}$$

Statement.

What is the individual portion of the present value ·0585 of £1, to be received by the only survivor at 96, if ·2412 is the present value of £1 to each of four survivors at 95?

$$\left.\begin{matrix} x \\ \cdot 2412 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 0585 \end{matrix}\right\} = \cdot 242.$$

TABLE IX

Value of an Annuity of £1 on a Single Life, at every Age, according to the Northampton Table of Mortality, at the several rates of 3, 4, and 5 per cent. per annum.

Age.	3 per Cent.	4 per Cent.	5 per Cent.	Age.	3 per Cent.	4 per Cent.	5 per Cent.
1	16·021	13·465	11·563	49	12·693	11·475	10·443
2	18·599	15·633	13·420	50	12·436	11·264	10·269
3	19·575	16·462	14·135	51	12·183	11·057	10·097
4	20·210	17·010	14·613	52	11·930	10·849	9·925
5	20·473	17·248	14·827	53	11·674	10·637	9·748
6	20·727	17·482	15·041	54	11·414	10·421	9·567
7	20·853	17·611	15·166	55	11·150	10·201	9·382
8	20·885	17·662	15·226	56	10·882	9·977	9·193
9	20·812	17·625	15·210	57	10·611	9·749	8·999
10	20·663	17·523	15·139	58	10·337	9·516	8·801
11	20·480	17·393	15·043	59	10·058	9·280	8·599
12	20·283	17·251	14·937	60	9·777	9·039	8·392
13	20·081	17·103	14·826	61	9·493	8·795	8·181
14	19·872	16·950	14·710	62	9·205	8·547	7·966
15	19·657	16·791	14·588	63	8·910	8·291	7·742
16	19·435	16·625	14·460	64	8·611	8·030	7·514
17	19·218	16·462	14·334	65	8·304	7·761	7·276
18	19·013	16·309	14·217	66	7·994	7·488	7·034
19	18·820	16·167	14·108	67	7·682	7·211	6·787
20	18·638	16·033	14·007	68	7·367	6·930	6·536
21	18·470	15·912	13·917	69	7·051	6·647	6·281
22	18·311	15·797	13·833	70	6·734	6·361	6·023
23	18·148	15·680	13·746	71	6·418	6·075	5·764
24	17·983	15·560	13·658	72	6·103	5·790	5·504
25	17·814	15·438	13·567	73	5·794	5·507	5·245
26	17·642	15·312	13·473	74	5·491	5·230	4·990
27	17·467	15·184	13·377	75	5·199	4·962	4·744
28	17·289	15·053	13·278	76	4·925	4·710	4·511
29	17·107	14·918	13·177	77	4·652	4·457	4·277
30	16·922	14·781	13·072	78	4·372	4·197	4·035
31	16·732	14·639	12·965	79	4·077	3·921	3·776
32	16·540	14·495	12·854	80	3·781	3·643	3·515
33	16·343	14·347	12·740	81	3·499	3·377	3·263
34	16·142	14·195	12·623	82	3·229	3·122	3·020
35	15·938	14·039	12·502	83	2·982	2·887	2·797
36	15·729	13·880	12·377	84	2·793	2·708	2·627
37	15·515	13·716	12·249	85	2·620	2·543	2·471
38	15·298	13·548	12·116	86	2·462	2·393	2·328
39	15·075	13·375	11·979	87	2·312	2·251	2·193
40	14·848	13·197	11·837	88	2·185	2·131	2·080
41	14·620	13·018	11·695	89	2·013	1·967	1·924
42	14·391	12·838	11·551	90	1·794	1·758	1·723
43	14·162	12·657	11·407	91	1·501	1·474	1·447
44	13·929	12·472	11 258	92	1·190	1·171	1·153
45	13·692	12·283	11·105	93	·839	·827	·816
46	13·450	12·089	10·947	94	·536	·530	·524
47	13·203	11·890	10·784	95	·242	·240	·238
48	12·951	11·685	10·616	96	·000	·000	·000

IMMEDIATE ANNUITIES

39°. What is the present value by Table IX. of an annuity of £40, payable yearly, during a life aged forty-five; interest 3 per cent.?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 40 \\ 13{\cdot}692 \end{matrix}\right\} = 547{\cdot}68.$$

Required the present value, by Table IX., of an annuity of £20, payable half-yearly, on a life aged thirty-five; interest 4 per cent.?

Present value, by Table IX., of annuity at 35 = 14·039
Add compensation for half-year's risk and interest .... ·25

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 20 \\ 14{\cdot}289 \end{matrix}\right\} = 285{\cdot}78.$$

Required the present value of an annuity, at 5 per cent., of £60, payable quarterly, on a life aged thirty.

Add ·375 to the tabular value for the quarterly payment

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 60 \\ 13{\cdot}447 \end{matrix}\right\} = 806{\cdot}82.$$

A gentleman aged forty-five, holding the lease of an estate during life, of the clear annual rent of £60, one year's payment of which is just due, wishes to dispose of the same; what is the value of the title, the rate of interest being 4 per cent.?

N.B. In calculating annuities with one year's payment, due at the *beginning* of the year, add unity to the tabular value of £1 annuity.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 60 \\ 9{\cdot}977 + 1 \end{matrix}\right\} = £658\ 12s.\ 5d.$$

WHAT ANNUITY A GIVEN SUM WILL PURCHASE.

40°. An annuity on a life aged forty-five, was purchased for the sum of £684 12*s*.; what was the amount of the annuity, allowing interest at 3 per cent.?

$$\left.\begin{matrix} x \\ 13{\cdot}692 \end{matrix}\right\} = \left\{\begin{matrix} 684{\cdot}6 \\ 1 \end{matrix}\right\} = £50.$$

An annuity, payable half-yearly, on a life aged twenty-seven, was purchased for the sum of £1362 14*s.*; what was the amount, at 5 per cent. interest?

$$\left.\begin{matrix} x \\ (13{\cdot}777 + {\cdot}25) \end{matrix}\right\} = \left\{\begin{matrix} 1362{\cdot}7 \\ 1 \end{matrix}\right\} = £100 \text{ per annum, payable half-yearly.}$$

DEFERRED ANNUITIES.

41°. The present value, or single premium for an annuity to be received at the expiration of a given time, equals the value of an annuity at the increased age, counted by the present value of £1, to be received at the end of the deferred term; but since the payment of the annuity is dependent on individual life, this present value must be counted; that is, diminished by the probability of the life attaining the increased age.

To find the annual premium, or the average value of the deferred annuity for one year of the term, deduct from £1, or unity, its present value if paid at the end of the deferred term, at the given rate of interest, convertible yearly, counted by the probability of the given life surviving that term. This product is the discount of £1 for the deferred term, diminished by the probability. Add this product to the difference between the values of an annuity at the present age and an annuity deferred to the increased age. The single premium, divided by this product, will give the annual premium.

It is obvious that the same rule will apply for the finding of the *half-yearly* or *quarterly* premiums, provided the present value of £1 be substituted, with the interest convertible at these periods, and one-half or one-fourth of the result taken.

Examples.

42°. What is the present value of the reversion of an annuity of £50, payable yearly, during the remainder of a life aged forty-five, after the age of fifty-five; rate of interest 3 per cent?

Statement.

Let r^d be the general expression for the present value of

£1 due at the end of any deferred term, a_m present value of an annuity of £1 at m years of age; $P_{m,n}$, probability of a life aged m living n years; then,

$$\left.\begin{array}{l} x \\ r^{10} \\ a_{55} \\ P_{45,10} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \cdot 744 \\ 11 \cdot 15 \\ \cdot 7537 \end{array}\right\} \times 50 = 312 \cdot 619, \text{ i.e., } £312\ 12s.\ 4\tfrac{1}{2}d.$$

What premium ought to be given now to secure an annuity of £67 19s. 9d., payable half-yearly, during a life now aged seventeen, after the next thirteen years; interest 4 per cent.?

$$\left.\begin{array}{l} x \\ r^{13} \\ a_{30} \\ P_{17,13} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \cdot 6006 \\ 14 \cdot 781 + 25 \\ \cdot 8242 \end{array}\right\} \times 67 \cdot 987 = £505\ 17s.\ 5d.$$

The single premium for a deferred annuity of £50, at 3 per cent., payable yearly, during the remainder of a life aged forty-five, after the age of fifty-five, is 312·619. What are the annual and half-yearly premiums, payable at the begin ning of each year and half-year?

$$1 - (\cdot 744 \times \cdot 7537) + (13 \cdot 692 - 6 \cdot 253) = 7 \cdot 889,$$

average for one year

$$\left.\begin{array}{r} x \\ 7 \cdot 889 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ 312 \cdot 619 \end{array}\right\} = 39 \cdot 6273, \text{ or } £39\ 12s.\ 6\tfrac{1}{2}d.,$$

annual premium.

$$(1 + \cdot 05)^{-20} = \cdot 61027,$$

present value of £1 in ten years, convertible half-yearly;

$$1 (\cdot 61027 \times \cdot 7537) + (13 \cdot 692 - 6 \cdot 253) = 7 \cdot 979,$$

average for two half-years.

$$\left.\begin{array}{r} x \\ 7 \cdot 979 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ 312 \cdot 619 \end{array}\right\} = \frac{39 \cdot 1802}{2} = £19\ 11s.\ 9\tfrac{3}{4}d.,$$

half-yearly premium.

The sum of £400 was paid for the purchase of a life annuity, at the age of thirty-six, to commence at the expiration of nine years; required the amount of the annuity, interest 4 per cent.

$$\left.\begin{matrix} x \\ r^9 \\ a_{45} \\ P_{36,9} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 7026 \\ 12\cdot 2835 \\ \cdot 8254 \end{matrix}\right\} = 7\cdot 1234 \text{ deferred annuity of £1.}$$

$$\frac{400}{7\cdot 1234} = £56\ 1s.\ 2d.$$

TEMPORARY ANNUITIES.

43°. The present value of a temporary annuity is equal to the difference between the present values of a deferred annuity of £1 on the given life, and an annuity for the whole term of life.

For temporary annuities payable at the *beginning* of each year, add unity to the present value of an annuity for one year less than the given term, payable at the *end* of each year.

What is the present value of an annuity payable yearly, for the next ten years, on a life aged forty-five; rate of interest 3 per cent.?

$$\left.\begin{matrix} x \\ r^{10} \\ a_{55} \\ P_{45,10} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 744 \\ 11\cdot 15 \\ 7537 \end{matrix}\right\} = 6\cdot 2523 \therefore \ldots\ldots \left\{\begin{matrix} 13\cdot 692 \ldots a_{45} \\ 6\cdot 2523 \quad a_{\overline{45}}|_{10} \\ \hline 7\cdot 44 \quad a_{\overline{45}|}^{10} \end{matrix}\right.$$

Notation

$$a_{\overline{m}}|_n$$

present value of an annuity deferred n years on a life aged m.

$$a_{\overline{m}|}^{\,n}$$

present value of an annuity for the next n years on a life aged m.

What premium ought to be given to secure an annuity of £67 19s. 9d., payable half-yearly, for the next thirteen years, provided a person aged seventeen survive so long; interest 4 per cent.?

$$\left.\begin{array}{l} x \\ r^{13} \\ a_{30} \\ P_{17,13} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \quad \cdot 6006 \\ 15\cdot 031 \text{ payable half-yearly} \\ \quad \cdot 8242 \end{array}\right\} \begin{array}{l} a_{\overline{17}|}{}_{13} \\ = 7\cdot 4409 \end{array}$$

$$\begin{array}{l} 16\cdot 712\ a_{17} \text{ payable half-yearly} \\ \ \ 7\cdot 4409\ a_{17|13} \\ \hline 9\cdot 271\ a_{\overline{17}|13} \times 67\cdot 987 = £630\ 6s. \end{array}$$

Required the present value of the lease of an estate for fifty years on the life of a boy aged twelve years, valued at £350 per annum, out of which the tenant has to pay various charges, to the amount of £65 annually; rate of interest 4 per cent.

N.B. In questions of this kind, reserved rent, tithe, taxes, and all other charges, must be deducted, to ascertain the *clear annual rent.*

$$\left.\begin{array}{l} x \\ r^{50} \\ a_{62} \\ P_{12,50} \end{array}\right\} = \left\{\begin{array}{l} 1\cdot \\ \ \ \cdot 1407 \\ 8\cdot 547 \\ \ \ \cdot 3363 \end{array}\right\} = \cdot 4044\ a_{\overline{12}|50}$$

$$\begin{array}{l} 17\cdot 251\ a_{12} \\ \quad \cdot 4044 \\ \hline 16\cdot 846\ a_{\overline{12}|50} \times (350 - 65) = £4801\ 5s\ 6d. \end{array}$$

DEFERRED TEMPORARY ANNUITIES.

44°. The present value of a deferred temporary annuity is the difference between an annuity deferred for the given term, and an annuity deferred for the given term *increased* by the period of the temporary annuity.

When a reversionary annuity is secured by an annual premium, the first payment is usually made immediately, and the subsequent payments at the end of each year, until the reversion is entered upon.

The divisor for the annual premium for a deferred temporary annuity of £1 paid down, and £1 at the end of each year for n years, equals the deferred annuity divided by unity, plus a temporary annuity for the deferred term.

$$\frac{a_{\overline{m}|n}}{1 + a_{\overline{m}|n}}$$

i.e., to find the annual premium necessary to secure an annuity for ten years, to be entered upon at the expiration of eight years, divide the present value of the deferred temporary annuity by unity, added to the present value of an annuity for eight years.

$$\frac{a_{\overline{m}|8}}{1 + a_{\overline{m}|8}}$$

What are the single and annual premiums for an annuity of £1, to be entered upon at the expiration of eight years, and then to continue ten years, subject to the existence of a life now aged forty-five; interest 4 per cent.?

$$\left.\begin{array}{l} x \\ r^{8} \\ a_{53} \\ P_{45,8} \end{array}\right\} = \left\{\begin{array}{r} 1\cdot \\ \cdot7307 \\ 10\cdot637 \\ \cdot804 \end{array}\right\} \begin{array}{l} a_{\overline{45}|8} \\ =6\cdot2504 \end{array}$$

$$\left.\begin{array}{l} x \\ r^{18} \\ a_{63} \\ P_{45,18} \end{array}\right\} = \left\{\begin{array}{r} 1 \\ \cdot4937 \\ 8\cdot291 \\ \cdot552 \end{array}\right\} \begin{array}{l} a_{\overline{45}|18} \\ =2\cdot2596 \end{array}$$

$$\begin{array}{l} 6\cdot2504 \; a_{\overline{45}|8} \\ 2\cdot2596 \; a_{\overline{45}|18} \\ \hline 3\cdot9908 \end{array}$$

3·9908 single premium for a_{45}, deferred 8 years, to continue 10 years.

$$\begin{array}{l} 12\cdot283 \; a_{45} \\ 6\cdot25 \;\; a_{\overline{45}|8} \\ \hline 6\cdot033 \; a_{\overline{45}|8} \end{array} \quad (1 + 6\cdot033) = 7\cdot033.$$

$\frac{3\cdot99}{7\cdot03} = \cdot576$, or 11*s.* 4½*d.*, annual premium.

INCREASING AND DECREASING ANNUITIES.

45°. The calculation of increasing and decreasing aunuities is somewhat complicated, and requires the construction of a preparatory table consisting of three columns, of which the first should contain the products of the number living at each age, multiplied respectively by the present values of £1 to be received at the end of the same number of years as the age; the second should contain the sum of the products of the same factors at all ages above; the third, the sum of the products of the same factors at the given age, *and* all ages above. Let these columns be distinguished as A, B, C, and let l_m represent the number living at the given age, r^m present value of £1 to be received at the end of m years.

The following is the general formula for the present value of an annuity of £1 at thirty years of age last birthday, increasing $\frac{1}{8}$ each year; interest 3 per cent.

$$\frac{1 - \cdot 125 \times \overset{\text{B}}{(l_{31} \times r^{31} + \text{all ages above})} + \cdot 125 \times \overset{\text{C}}{(l_{30} \times r^{30} + \text{\&c.})}}{\overset{\text{A}}{(l_{30} \times r^{30})}}$$

If the annuity be taken for £40, increasing £5 each year, the formula becomes,

$$\frac{40 - 5 \times (l_{31} \times r^{31} + \text{\&c.}) + 5 \times (l_{30} \times r^{30} + \text{\&c.})}{(l_{30} \times r^{30})},$$

which is the same as,

$$\left(\frac{l_{31} \times r^{31} + \text{\&c.}}{l_{30} \times r^{30}}\right) \times 35 + \left(\frac{l_{30} \times r^{30} + \text{\&c.}}{l_{30} \times r^{30}}\right) \times 5$$

that is, $(a_{30} \times 35) +$ &c., or, the present value of an immediate annuity of £35 added to the present value of an annuity at the same age of £5, increasing £5 every year. The following exhibits the principle more plainly:—

$$1^\circ \text{ payment is increased by } \frac{r^{30} \times l_{30}}{r^{30} \times l_{30}} \times 5 = £5$$

$$35 + 5 = £40$$

2° payment is increased by $\frac{r^{31} \times l_{31}}{r^{30} \times l_{30}} \times 5$, present value of a_{30} of £5 for two years. $35 + 10$.

3° payment is increased by $\frac{r^{32} \times l_{32}}{r^{30} \times l_{30}} \times 5$, present value of a_{31} of £5 for three years. $35 + 15$, &c. &c.

When the quantities A, B, C, are previously calculated, the terms may be stated in two equations; the first being simply the usual form of finding the value of an annuity of £35, by Table IX.

$$\left.\begin{matrix} x \\ a_{30} \end{matrix}\right\} = \left\{\begin{matrix} 35 \\ 16{\cdot}922 \end{matrix}\right\} = 592{\cdot}27$$

$$\left.\begin{matrix} x \\ C \\ 1806{\cdot}562 \end{matrix}\right\} = \left\{\begin{matrix} 5 \\ 446138{\cdot}7 \\ \text{A} \end{matrix}\right\} = 1234{\cdot}77,$$

and 592·27
1234·77

1827·04 present value of a_{30} of £40, increasing £5 yearly.

For the same annuity *decreasing* £5 every year, the formula becomes:—

$$a_{30} \times 40 + 5 - \left(\frac{l_{30} \times r^{30} + \&c.}{l_{30} \times r^{30}}\right) \times 5.$$

As in the former example, the first payment being exactly £40, the amount of *decrease*, 5, must be *added* to the annuity to compensate for its deduction from the first payment.

TEMPORARY INCREASING AND DECREASING ANNUITIES.

46°. The following is the formula for an annuity of £40, on a life aged thirty, for ten years, *increasing* £5 every year.

$$a_{\overline{30}|\atop 10|} \times \frac{(40-5)+(l_{30} \times r^{30} + \&c. - l_{40} \times r^{40} + \&c.).5 - (l_{41} \times r^{41} + \&c.).5}{(l_{30} + r^{30})}.$$

For the same annuity *decreasing* £5 each year, the formula becomes—

$$a_{\overline{30}|_{10}} \times \frac{(40+5)-(l_{30}\times r^{30}+\&c.-l_{40}\times r^{40}+\&c.).5-(l_{41}\times r^{41}+\&c.).5}{(l_{30}\times r^{30})}.$$

PREPARATORY TABLE FOR THE CALCULATION OF ANNUITIES.

47°. The following statement, taken at the oldest age but three, presents, at a glance, the method of constructing the Preparatory Table in Jones's work on Annuities, so far as the first three columns. Each column contains opposite each age, the sum of the products of the following factors :—

Age.	A.	B.	C.
93	$l_{93}\times r^{93}$	$l_{94}\times r^{94}$ $l_{95}\times r^{95}$ $l_{96}\times r^{96}$	$l_{93}\times r^{93}$ $l_{94}\times r^{94}$ $l_{95}\times r^{95}$ $l_{96}\times r^{96}$

ENDOWMENTS

48°. To find the present value of a sum of money to be received at the end of a given number of years, dependent on individual life.

The present value of £1 due at the end of the term, must be counted by the probability that the given life will survive the term.

When the endowment is secured by an annual payment, divide the single premium by a temporary annuity for one year less than the deferred term; but as the first payment is usually made immediately, add *unity* to the temporary annuity before dividing.

If the premium be payable half-yearly, or quarterly, add ·25, or ·375 in finding the value of the deferred annuity, and again before subtracting for the temporary annuity.

Examples

Required the present value, in a single payment of £300,

to be received at the end of fifteen years, provided a person now aged twenty, shall be then living; interest 4 per cent.

$$\left.\begin{matrix} x \\ r^{15} \\ P_{20,\,15} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 555 \\ \cdot 7814 \end{matrix}\right\} \times 300 = £130\ 2s.\ 0\tfrac{1}{4}d.$$

TABLE X.

Value of the Reversion, or the Single Premium for the Assurance of £1, payable at the end of the Year in which a given Life shall fail, deduced from the Northampton Table of Mortality, at the several rates of 3, 4, and 5 per cent. per annum.

Age.	3 per Cent.	4 per Cent.	5 per Cent.	Age.	3 per Cent.	4 per Cent.	5 per Cent.
15	·39833	·31573	·25771	45	·57207	·48912	·42357
16	·40480	·32212	·26381	46	·57912	·49658	·43110
17	·41112	·32838	·26981	47	·58632	·50423	·43886
18	·41709	·33427	·27538	48	·59366	·51212	·44686
19	·42271	·33973	·28057	49	·60117	·52019	·45510
20	·42801	·34488	·28538	50	·60866	·52831	·46338
21	·43291	·34954	·28967	51	·61603	·53627	·47157
22	·43754	·35396	·29367	52	·62339	·54427	·47976
23	·44229	·35846	·29781	53	·63085	·55242	·48819
24	·44709	·36308	·30200	54	·63842	·56073	·49681
25	·45201	·36777	·30633	55	·64611	·56919	·50562
26	·45702	·37262	·31081	56	·65392	·57781	·51462
27	·46212	·37754	·31538	57	·66181	·58658	·52386
28	·46731	·38258	·32010	58	·66979	·59554	·53329
29	·47261	·38777	·32490	59	·67792	·60462	·54290
30	·47799	·39304	·32990	60	·68610	·61388	·55276
31	·48353	·39850	·33500	61	·69438	·62327	·56281
32	·48912	·40404	·34029	62	·70276	·63281	·57305
33	·49486	·40973	·34571	63	·71136	·64265	·58371
34	·50071	·41558	·35129	64	·72007	·65269	·59457
35	·50666	·42158	·35705	65	·72901	·66304	·60590
36	·51274	·42769	·36300	66	·73804	·67354	·61743
37	·51898	·43400	·36910	67	·74712	·68419	·62919
38	·52530	·44046	·37543	68	·75630	·69500	·64114
39	·53179	·44712	·38195	69	·76550	·70588	·65329
40	·53840	·45396	·38871	70	·77474	·71688	·66557
41	·54504	·46085	·39548	71	·78394	·72788	·67790
42	·55171	·46777	·40233	72	·79311	·73885	·69029
43	·55838	·47473	·40919	73	·80211	·74973	·70262
44	·56517	·48185	·41629	74	·81094	·76038	·71476

A gentleman, aged thirty-six, will be entitled to receive the sum of £1000 at the expiration of fifteen years, provided

F

he survive the term. What annual premium, payable half-yearly, ought he to receive in lieu of it, reckoning interest 5 per cent. per annum?

$$\left.\begin{matrix} x \\ r^{15} \\ P_{36,\,15} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 481 \\ \cdot 70546 \end{matrix}\right\} = \cdot 339326 \times 1000 = £339\cdot 326,$$

present value of endowment in a single payment.

$$\left.\begin{matrix} & x \\ & r^{14} \\ a_{50} & + \cdot 25 \\ & P_{36,\,14} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 505 \\ 10\cdot 519 \\ \cdot 726 \end{matrix}\right\} = 3\cdot 8573,$$

annuity at 36, deferred fourteen years, payable half-yearly.

$$\begin{array}{ll} a_{\overline{36|}\,14} & 12\cdot 377\ a_{36} \\ (3\cdot 8573 + \cdot 25) = & 4\cdot 107 \\ \hline & 8\cdot 269 = a_{\overline{\substack{36 \\ 14}|}} \text{ payable half-yearly.} \end{array}$$

$$(8\cdot 269 + 1) = \left.\frac{339\cdot 326}{9\cdot 269}\right\} = 36\cdot 6087, \textit{ i. e.}, £36\ 12s.\ 2d.,$$

annual premium, payable half-yearly.

General Formula.

To find the annual premium necessary to secure a sum of money payable in n years. Let s equal the present value of the endowment, then

$$\left.\frac{s}{a_{\overline{\substack{m \\ n-1}|}}}\right\} = \text{annual premium.}$$

ASSURANCES ON LIVES.

49°. A life assurance is an engagement to secure the payment of a sum on the death of an individual, in consideration of a stipulated single or annual payment.

The present value of a perpetuity, or the sum that will produce a perpetual annuity of £1, at 3 per cent., is $\frac{1}{\cdot 03}$, or 33·33. If, from this perpetuity we deduct 13·45, the present

value of an immediate annuity at forty-six, the difference, 19·883, is the present value of the perpetuity of £1, to be received on the death of a life aged forty-six. If this be divided by the present value of a perpetuity to be received immediately, or the principal that will produce £1 per annum for ever, it will give the principal that will produce £1 at the end of the life, or the present value of the reversion of £1; but, as the premium is usually paid at the time of effecting the assurance, add unity to the perpetuity before dividing.

By this method, Table X. may be constructed, which gives the present values of the reversion of £1, at various ages, at 3, 4, and 5 per cent., according to the Northampton table of mortality.

The annual premium is an annuity, of which the first payment is made in advance; therefore, the single premium, divided by an annuity on the given life, plus 1, equals the annual premium. If the premium be payable half-yearly, deduct ·25; if quarterly, ·375, from the annuity.

The annuity of £1 at the same age being calculated on the same probability, is evidently a measure of its annual value; but, as the annuity ceases at death, while the reversion is usually paid at the end of the year in which the life may fail, the annuity is increased by unity before dividing.

Examples.

50°. Required the premium in a single payment, to assure £1 at the death of a person aged fifteen, reckoning interest 4 per cent

Statement.

What is the present value of the reversion of £1, if 25 + 1 is the present value of £1 per annum for ever, to be received immediately, the first payment in advance; and £25 — 16·791 a_{15}, *i. e.*, 8·209, is the present value of the same perpetuity to commence on the decease of a life aged fifteen?

$$\left.\begin{matrix} x \\ 26 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 8{\cdot}209 \end{matrix}\right\} = {\cdot}31573,$$

present value of the reversion of £1 on a life aged 15.

What premium ought to be paid for the assurance of £500 on a life aged forty-five, to be received at the end of the year in which the life shall fail, interest 4 per cent.?

Let R_m equal the value of the reversion of £1, at m years of age.

$$\left.\begin{array}{r} x \\ R_{45} \end{array}\right\} = \left\{\begin{array}{l} 500 \\ \qquad \cdot48912 \end{array}\right\} = \text{£}244\ 11s.\ 2\tfrac{1}{4}d.$$

Required the annual premium, payable yearly and half-yearly, for the assurance of £100 on a life aged forty-three, interest 3 per cent.

$$\left.\begin{array}{r} x \\ R_{43} \\ 15\cdot162 \end{array}\right\} = \left\{\begin{array}{l} 100 \\ \qquad \cdot55838 \\ (a_{43} + 1) \end{array}\right\} = \text{£}3\ 13s.\ 7\tfrac{3}{4}d.,\ \text{annual premium.}$$

$$\left.\begin{array}{r} x \\ R_{43} \\ 14\cdot912 \end{array}\right\} = \left\{\begin{array}{l} 100 \\ \qquad \cdot55838 \\ (a_{43} + 1 - \cdot25) \end{array}\right\} = \text{£}3\ 14s.\ 10\tfrac{1}{2}d.,$$

annual premium, payable half-yearly; £1 17s. $5\tfrac{1}{4}d.$, half-yearly payment.

A person aged forty-two pays an immediate sum of £1000, and an annual premium for the assurance of £5000 on his life. What annual premium will be required, rate of interest 3 per cent.?

$$\left.\begin{array}{r} x \\ R_{42} \end{array}\right\} = \left\{\begin{array}{l} 5000 \\ \qquad \cdot55171 \end{array}\right\} =$$

2758·55 single premium to assure £5000
1000 deduct present payment.

1758·55 ÷ 15·391, $a_{42} + 1 = \text{£}114\ 5s.\ 2d.$, ann. premium

What single premium is equal to £1 annual premium, for the assurance of a life aged forty-five, interest 3 per cent.?

$$\left.\begin{array}{r} x \\ a_{45} + 1 \end{array}\right\} = \left\{\begin{array}{c} 1 \\ 14\cdot692 \end{array}\right\} = 14\cdot692.$$

The reversion of £100, secured by a policy of assurance on a life aged forty-five, was purchased for the sum of £57 4s. 2d. Required the annual premium, interest 3 per cent.

$$\left.\begin{array}{r} x \\ 14\cdot692 \end{array}\right\} = \left\{\begin{array}{c} 57\cdot2075 \\ (a_{45} + 1) \end{array}\right\} = \text{£}3\ 17s.\ 10\tfrac{1}{2}d.$$

ASSURANCES WITH A LIMITED NUMBER OF ANNUAL PREMIUMS.

51°. We have seen that the present value of a reversion must be measured by an annuity on the given life, plus 1, to find its annual premium. On the same principle, the divisor for the annual premium for a *limited* number of years, is a *temporary* annuity, increased by unity for the same term But since the first premium is paid in advance, and the remainder at the end of each year, the annuity must be taken for one year less than the given term, according to the following formula:—

$$\frac{R_m}{a_{\overline{m}|_{n-1}} + 1} = \text{annual premium, limited to } n \text{ payments.}$$

What premium, limited to ten annual payments, the first immediately, ought to be given for a policy of assurance for £100, on a life aged forty-five, the payments to cease in case of death before the end of the term; interest 3 per cent.?

Statement.

What is the annual value, limited to ten payments, of a reversion, if an annuity at forty-five, for nine years, increased by unity, be the present value of £1 at the end of each year of the term?

$$\left.\begin{matrix} x \\ r^9 \\ a_{54} \\ P_{45,9} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \cdot 7664 \\ 11\cdot 414 \\ \cdot 7789 \end{matrix}\right\} = 6\cdot 814\, a_{\overline{45}|9} \qquad \begin{array}{r} 13\cdot 692\, a_{45} \\ 6\cdot 814 \\ \hline 6\cdot 878\, a_{\overline{45}|_9} \end{array}$$

$$\frac{\cdot 57207\, R_{45}}{7\cdot 878\, (a_{\overline{45}|_9} + 1)} = \cdot 07262 \times 100 = 7\cdot 262,$$

annual premium for £100.

DEFERRED ASSURANCES.

52°. If the assurance is to commence after a term of years, the value of the reversion at the increased age must be counted by the probability of the given life surviving that term, and by the present value of £1 to be received at the end of the term.

The *annual* premium, when the first payment is made immediately, is obtained by dividing the single premium by a temporary annuity increased by unity, for one year less than the number of years deferred.

Examples

Required the value of an assurance deferred eleven years, on a life aged forty-five; rate of interest 3 per cent.

$$\left.\begin{array}{r} x \\ R_{45+11} \\ P_{45,\ 11} \\ r^{11} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \cdot 6539 \\ \cdot 7284 \\ \cdot 7224 \end{array}\right\} = \cdot 34412,$$

present value of reversion of £1 deferred eleven years.

$$\frac{\cdot 34412}{8\cdot 44\ (a_{\overline{\frac{45}{10}}|} + 1)} = \cdot 040778, \text{ annual premium.}$$

A sum payable at the death of a person aged sixty, and subject to his surviving ten years beyond that age, amounted to £900; required what that sum was, supposing the rate of interest 5 per cent.

$$\left.\begin{array}{r} x \\ R_{70} \\ P_{60,\ 10} \\ r^{10} \end{array}\right\} = \left\{\begin{array}{l} 900 \\ \quad \cdot 6656 \\ \quad \cdot 6045 \\ \quad \cdot 6139 \end{array}\right\} = £222\cdot 305.$$

TEMPORARY ASSURANCES.

53°. A temporary assurance is the difference between an assurance for the whole term of life and an assurance deferred for the given period.

As in the last example, the divisor for the annual premium is the present value of a temporary annuity increased by unity, for one year less than the given term.

If the assurance be required for one year only, multiply the present value of £1, due one year hence, by the probability that the given life will die in the year.

The following is the general formula for an assurance on a life aged m, for the term of n years.

$$(R_m - R_{\overline{m}|n}) = R_{\overline{m}|_n} \qquad \left.\frac{R_{\overline{m}|_n}}{(a_{\overline{m}|_{n-1}} + 1)}\right\} = \text{annual premium.}$$

Examples.

Required the value of the reversion of £1, on a life aged forty-seven, for seven years hence; interest 5 per cent.

$$\left.\begin{array}{r} x \\ R_{54} \\ r^7 \\ P_{47,7} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \;\cdot 49681 \\ \;\cdot 710681 \\ \;\cdot 81824 \end{array}\right\} = \cdot 28889, \text{ reversion at forty-seven, deferred seven years.}$$

$\cdot 43886\ R_{47}$
$\cdot 28889\ R_{\overline{47}|7}$

$\cdot 14996\ R_{\overline{47}|_7}$ = present value of reversion of £1, for 7 years.

Required the annual premium to assure the sum of £100 on a life aged forty-five, in case of death during the next eleven years; interest 3 per cent.

$$\left.\begin{array}{r} x \\ R_{56} \\ r^{11} \\ P_{45,\,11} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \;\cdot 65392 \\ \;\cdot 7224 \\ \;\cdot 72845 \end{array}\right\} = \cdot 34412\ R_{\overline{45}|11}$$

$\cdot 57207$
$\cdot 34412$ deduct.

$\cdot 22795 = R_{\overline{45}|_{11}}$ *i. e.*, present value of reversion of £1 for eleven years.

$$\left.\begin{array}{r} x \\ r^{10} \\ a_{57} \\ P_{47,\,10} \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \;\cdot 744 \\ 10\cdot 611 \\ \;\cdot 73867 \end{array}\right\} = 5\cdot 832\ a_{\overline{47}|10}$$

$13\cdot 203$
$5\cdot 832$

$7\cdot 371\ a_{\overline{47}|_{10}}$ temporary annuity at forty-seven, for ten years.

$$\left.\frac{\cdot 22795\ R_{\overline{45}|\atop 11}}{\left(7\cdot 371\ a_{\overline{45}|\atop 10} - 1\right)}\right\} = \cdot 02723 \times 100 = 2\cdot 723 \left\{\begin{array}{l}\text{annual premium}\\ \text{for temporary as-}\\ \text{surance of £100,}\\ \text{at forty-five.}\end{array}\right.$$

ASSURANCES FOR ONE YEAR.

54°. The value of an assurance for one year is equal to the present value of £1 to be received at the end of the year, counted by the probability of the given life failing to survive one year.

Example.

Required the premium for the assurance of £100, for one year, on a life aged twenty; interest 3 per cent.

Probability, by Table VII., of a life of twenty failing in one year $= \frac{72}{5132}$, or ·0140296.

$$\left.\begin{array}{r} x \\ P_{20,1} \\ r^1 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ \cdot 014029 \\ \cdot 97087 \end{array}\right\} = \cdot 0136238 \times 100 = 1\cdot 362, \text{ or } £1\ 7s.\ 3d.$$

VALUE OF POLICIES.

55°. Since a policy of assurance secured by a periodical premium consists of an annuity paid to the office, for the expectation of a given sum at the decease of the person assured, the seller of a policy exonerates himself from all future payments of the annuity, and also abandons the benefit of the reversion. Therefore, supposing the annual premium *just due and not paid*, from the value of the reversion at the increased age, subtract the present value of an annuity, plus one, at the increased age, multiplied by the annual premium paid at the commencement of the policy. Thus, the present value of the reversion will be reduced by a sum equal to the present value of all the future premiums.

If, on the other hand, the premium *has been just paid*, we must add its amount to the above value.

Example.

What is the present value of a policy of assurance of £100 on a life aged forty-five, and which has been ten years in

existence; the annual premium, which is just due, being £3 17*s.* 10½*d.*; rate of interest 3 per cent.?

$$\left.\begin{array}{r} x \\ (a_{55}+1) \end{array}\right\} = \left\{\begin{array}{l} 3\cdot 8937 \\ 12\cdot 15 \end{array}\right\} = \text{'}47\cdot 3089)$$

$$\begin{array}{l} 64\cdot 611 \ \text{R}_{55} \times 100 \\ 47\cdot 308 \\ \hline 17\cdot 303 \end{array}$$ *i. e.*, £17 6*s.* 0¾*d.*, value of policy.

56°. *Value of a Bonus, or Addition to a Policy.*

Multiply the amount of the bonus by the present value of a reversion at the given age.

Example.

Required the present value of £240 added to a policy on a life now aged fifty; interest 3 per cent.

$$\left.\begin{array}{r} x \\ \text{R}_{50} \end{array}\right\} = \left\{\begin{array}{l} 240 \\ \quad\cdot 6086 \end{array}\right\} = £146 \ 1s. \ 6d.$$

57°. *What reduction of Annual Premium is equivalent to a given Bonus.*

Deduct from the annual premium the annual premium that will assure the amount of the bonus at the increased age.

Example.

What reduction of annual premium is equivalent to a bonus of £50, on a life now aged thirty-three; rate of interest 5 per cent.?

$$\left.\begin{array}{r} x \\ \text{Annual premium} \\ \text{for £1 at 33} \end{array}\right\} = \left\{\begin{array}{l} 50 \\ \quad\cdot 02516 \end{array}\right\} = £1 \ 5s. \ 2d.,$$

equivalent sum to deduct from annual premium.

PART III.

JOINT AND SURVIVORSHIP ANNUITIES AND REVERSIONS.

58°. The calculation of joint and survivorship annuities requires a voluminous series of tables, adapted to every difference in ages. These tables, being of very limited interest, and withal too large for publication at a moderate price, are usually kept in manuscript at the various life assurance offices. They are all calculated on the principles elucidated in the preceding portions of this work. It is, therefore, determined to give formulæ for the most useful cases, by a method of notation not quite so abstract as that of pure algebra, rather than swell the size of the volume with specimen tables, for the mere purpose of working exercises.

PROBABILITIES ON JOINT LIVES.

59°. To find the probability that two given lives shall both survive a certain number of years, or attain a given age.

Required the probability of two persons aged twenty and thirty respectively both surviving ten years.

$$\frac{l_{(20+10)}}{l_{20}} \times \frac{l_{(30+10)}}{l_{30}},\ i.\ e.,\ (P_{20,\,10} \times P_{30,\,10}) = P_{20,\,30,\,10}$$

joint probability of lives aged twenty and thirty surviving ten years.

To find the probability that the joint existence of two given lives shall fail in a certain number of years.

Required the probability that two given lives, aged twenty and thirty, shall *fail* within ten years.

Unity expresses certainty; therefore,

$$(1 - P_{20\,.\,30,\,10}) = P_{20\,.\,30,\,10}{}^{dd} \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\}$$ Probability of death within ten years, on two lives aged twenty and thirty

To find the probability that two given lives shall survive a given term.

$$(1 - P_{20.30,\,10}{}^{dd}) = P_{20.30,\,10}{}^{ll} \left\{ \begin{array}{l}\text{probability of both lives}\\ \text{surviving ten years.}\end{array}\right.$$

To find the probability that of two given lives *only one* of them shall survive a given term.

$$(P_{20,\,10} + P_{30,\,10}) - 2\,(P_{20.30,\,10}) = P_{20.30,\,10}{}^{\cdot},$$

probability that one of two lives aged 20 and 30 shall live ten years.

To find the probability that *one* of two given lives shall survive, and that the other shall die in a given term.

What is the probability that a life aged twenty shall survive, and that another aged thirty shall die in ten years?

$$P_{20,\,10.} \times \left(\frac{l_{30} - l_{40}}{l_{30}}\right), \text{ or } P_{30,\,10}{}^{d} = P_{20.30,\,10}{}^{l\;\;d},$$

probability that age 20 shall live and 30 shall die within ten years.

ANNUITIES ON JOINT LIVES.

60°. The following statement from the Northampton table may serve as a general formula for the construction of tabular values of annuities on joint lives.

Required the present value of an annuity on two joint lives, aged eighty-five and ninety.

Let r^{91}, as before, represent present value of £1, to be received at the end of 91 years, at the given rate per cent., then:

$$\left.\frac{r^{91} \times l_{91} \times l_{86} + \text{all ages above } 91}{r^{90} \times l_{90} \times l_{85}}\right\} = a_{85.90}$$

immediate annuity on joint lives 85 and 90.

Required, the present value of an annuity on two joint lives aged twenty-eight and thirty-three, deferred seven years.

$$(r^{7} \times a_{28.33.} \times P_{28,7.} \times P_{33,7.}) = a_{28.\overline{33}|_{7}}$$

annuity on joint lives 28 and 33, deferred 7 years

Value of a Deferred Annuity in annual payments.

Required the annual premium payable at the beginning of each year for an annuity on two joint lives, aged twenty-eight and thirty-three deferred seven years.

$$\frac{a_{28\,.\,\overline{33}|_7}}{(r^7 + P_{28,\,7} \times P_{33,\,7}) - (1 + a_{28.\,33} - \overline{a}_{28.\,\overline{33}|_7}} =$$

annual premium.

Required the present value of a temporary annuity, for the next ten years, on two joint lives, aged respectively twenty-seven and thirty-two.

$$(a_{27\,.\,32}) - (a_{27\,.\,\overline{32}|_{10}}) = a_{27\,.\,\overline{\overset{32}{10}}|}$$

annuity on joint lives aged 27 and 32 for 10 years

ASSURANCES FOR THE JOINT EXISTENCE OF TWO LIVES.

61°. Required the present value, in a single payment, of an assurance of the reversion of £1, to be received at the end of the year in which either of two given lives, say *ten* and *fifteen*, may fail.

Let p represent present value of a perpetuity of **£1,** and R present value of the reversion of £1 at the given rate per cent., then:

$$\frac{p - a_{10\,.\,15}}{p + 1} = R_{10\,.\,15},$$

value in a single payment of the reversion of £1, to be received on one of two lives aged 10 and 15.

$$\left.\frac{R_{10\,.\,15.}}{a_{10\,.\,15} + 1}\right\} = \text{annual premium for the same.}$$

What is the present value of a deferred assurance on the survivor of two lives, aged respectively twenty-seven and thirty-two, provided they both survive the term of eleven years?

$$R_{38.43.} \times P_{38.43,11.} = R_{38.\overline{43}|_{11}}$$

present value of reversion on lives 38 and 43, deferred 11 years.

$$\left.\frac{R_{38.\overline{43}|_{11}}}{a_{38.\overline{43}|_{11-1}} + 1}\right\} \text{annual premium for the same.}$$

Required the present value in a single payment of a temporary assurance for ten years on two joint lives, aged thirty-three and thirty-eight.

$$(R_{33.38}) - (R_{33.38|_{10}}) = R_{33.\overline{38}|_{10}}$$

single premium for ten years' assurance on lives 33 and 38.

$$\left.\frac{R_{33.38}}{a_{33.\overline{38}|_{10-1}}}\right\} = \text{annual premium for the same.}$$

Required the premium for a reversion on two lives aged twenty and twenty-five, on the contingency of their both surviving one year.

$$r^1 \times (P_{20.25,1}{}^{dd}) = R_{20.\overline{25}|_1}$$

present value of reversion for one year on lives 20 and 25.

ENDOWMENTS ON JOINT LIVES.

62°. Required the value, in a single payment, of an endowment to be received at the end of fifteen years, provided two persons aged twenty and twenty-five shall then be both living.

$$r^{15} + (P_{20.25,15}) = E_{20.\overline{25}|_{15}}$$

present value of endowment deferred fifteen years.

$$\frac{E_{20.\overline{25}|_{15}}}{a_{20.\overline{25}|_{15-1}}} = \text{annual premium for the same.}$$

VALUE OF POLICIES, ETC., ON JOINT LIVES.

63°. Required the present value of a policy of assurance effected on two joint lives, when at the ages of thirty-five and forty, now ten years of age.

Let A represent annual premium paid at the commencement of the policy, then:

$$A_{35\,.\,40} \times (a_{45\,.\,50} + 1)\ R_{45\,.\,50} = \text{present value.}$$

Required the present value of a bonus added to a policy on two joint lives, now aged forty-eight and fifty-three

Let B represent the amount of bonus, then:

$$R_{48\,.\,53} \times B = \text{present value.}$$

A policy of assurance was effected on two joint lives, when at the ages of thirty-four and thirty-nine respectively, to which at the expiration of ten years a reversionary bonus was added. What reduction in annual premium is the equivalent value of the bonus?

$$A_{44\ 49} \times B = \text{equivalent reduction.}$$

SURVIVOR OF TWO LIVES.

64°. Probability that of two given lives the survivor of them will outlive a certain number of years.

Required the probability that the survivor of two lives, whose respective ages are eighteen and twenty-three, will outlive the term of ten years.

$$(P_{18\,.\,10.} + P_{23,\,10}) = P_{\overline{18\,.\,23},\,10}^{\;v}$$

probability that of two lives, 18 and 23, survivor shall outlive ten years.

Required the probability that the survivor of two lives, whose respective ages are eighteen and twenty-three, shall *fail* to outlive the term of ten years.

$$(P_{18,\,10}{}^{d} + P_{23\ 10}{}^{d}) - (P_{18\,.\,23,\,10.}{}^{dd}) = P_{\overline{18\,.\,23},\,10}^{\;v\;\;d}$$

probability that of two lives 18 and 23, survivor shall die within ten years.

ANNUITIES ON TWO LIVES AND THE SURVIVOR.

65°. Required the present value of an annuity to continue during two lives and the survivor of them.

Let the respective ages be fifteen and twenty, then:

$$(a_{15} + a_{20}) - (a_{15.20}) = a_{\overline{15.20}}^{\;v}$$

annuity on both lives and the survivor

What are the values in single and annual payments of an annuity deferred twenty years, on two lives and the survivor, ages seventeen and twenty-two?

$$(a_{\overline{17}}|_{20} + a_{\overline{22}}|_{20}) - a_{17.\overline{22}}|_{20} = a_{17.\overline{22}}^{\;v}|_{20} \quad \text{single premium.}$$

$$\frac{a_{17.\overline{22}}^{\;v}|_{20}}{(r^{20} \times P_{22,\,20.}) - (1 + a_{22} - a_{\overline{22}}|_{20.})} \quad \text{annual premium.}$$

Required the present value of a temporary annuity, should both or either of two lives, aged eighteen and twenty-five, live twelve years.

$$(a_{\overline{18.25}}^{\;v}) - (a_{18.\overline{25}}^{\;v}|_{12}) = a_{\overline{18.25}}^{\;v\;\;v}|_{12}.$$

ASSURANCES ON THE LAST OF TWO LIVES.

66°. What present sum is required to assure £1, to be received at the end of the year in which the last of two lives, aged nineteen and twenty-four, may die?

$$\frac{p - a_{\overline{19.24}}^{\;v}}{p+1} = R_{\overline{19.24}}^{\;d}, \text{ single premium.}$$

$$\frac{R_{\overline{19.24}}^{\;d}}{(a_{\overline{19.24}}^{\;v} + 1)} = \text{annual premium.}$$

To find the value of the same, in annual payments, for a given number of years only.

Let n represent the given term of years.

$$\left.\frac{R_{\overline{19\,.\,24}}^{\,d}}{a_{\overline{19\,.\,24}|\,n-1}^{\,d}}\right\} = \text{annual premium for } n \text{ years.}$$

Let the ages be eighteen and twenty, and the assurance on the *last* of the two lives deferred ten years, then:

$$\left(R_{\overline{18}|10} + R_{\overline{20}|10}\right) - \left(R_{\overline{18\,.\,20}|10}\right) = R_{18\,.\,\overline{20}|10}^{\,d}$$

single premium.

$$\frac{R_{\overline{18\,.\,20}|10}^{\,d}}{a_{\overline{18\,.\,20}|\,10-1}^{\,d}} = \text{annual premium.}$$

Again, let the ages be eighteen and twenty, and the assurance as above, but temporary, for the term of ten years, then

$$\left(R_{\overline{18\,.\,20}}^{\,d}\right) - \left(_{\overline{18\,.\,20}|10}^{\,d}\right) = R_{\overline{18\,.\,20}|10}^{\,d}$$

When the reversion is for *one* year only, the present value is

$$\left(r^1 \times P_{18\,.\,20}^{\,d}\right) = R_{\overline{18\,.\,20}|1}^{\,d}$$

ENDOWMENTS.

67°. Required the value of an endowment on two lives, aged eighteen and twenty, to be received at the end of ten years, provided both or either survive that term.

$$\left(r^{10} \times P_{\overline{18\,.\,20,\,10}}^{\,v}\right) = \text{E, present value in single payment.}$$

$$\frac{E}{a_{\overline{18\,.\,20}|9}^{\,v}} = \text{annual premium.}$$

VALUE OF POLICIES ON LAST OF TWO LIVES.

68°. Required, the present value of a policy of assurance effected on the last survivor of two lives, at the respective ages of forty-seven and fifty-two, now ten years ago.

$$A_{47.52} \times \left(a_{\overline{57.62}}^{\,v} + 1\right) - R_{\overline{57.62}}^{\,v} = \text{value of policy}$$

$$\left(B \times R_{\overline{57.62}}^{\,v}\right) = \text{value of bonus on same policy.}$$

$$A_{\overline{57.62}}^{\,v} \times B) = \text{equivalent reduction in annual premium.}$$

SURVIVORSHIP ANNUITIES.

69°. Required the value of an annuity on a life aged twenty-five, to commence after the death of another person of the same age.

$$\left(a_{25} - a_{25.25}\right) = \text{single premium, or } a_{25|25}{}^{d}.$$

$$\frac{a_{25|25}{}^{d}}{\left(a_{25.25} + 1\right)} = \text{annual premium.}$$

To find the value, after the extinction of the existing life or lives, of the next in succession to be then nominated to an annuity to continue—

1°. During a succeeding life, $R_{59} \times (a_8 + 1) =$

$a_{8|59}{}^{d}$, annuity at 8 years of age, after death of 59.

2°. During joint existence of two succeeding lives, $R_{59} \times (a_{8.25} + 1) =$

$a_{8.25|59}{}^{d}$, annuity on joint lives 8 and 25, after death of 59.

3°. During the last survivor of two lives, $R_{59} \times (a_{\overline{8.25}}^{\,v} + 1) =$

$a_{\overline{8.25}|59}^{\,v}$ annuity on last survivor of two lives, 8 and 25, after death of age 59.

To find the present value of an annuity held during a given life, and also afterwards; namely, to commence at the end of the year in which the given life may fail—

1°. During another single life, $R_{60} \times (a_{25} + 1) =$

$a_{60|25}$, annuity on life of 60, and then on a life of 25.

2°. During two joint lives, $R_{60} \times (a_{8.25} + 1) =$

$a_{60|8.25}$ annuity on life of 60, and then on joint lives 8 and 25.

3°. During last survivor of two lives, $R_{60} \times (a_{\overline{8.25}}^{v} + 1) =$

$a_{60|\overline{8.25}}^{v}$ annuity on life of 60, and then on last survivor of two lives, 8 and 25.

To find the present value of an annuity during the existing life or lives, and afterwards for a term of years certain.

Annuity on a life of fifty, and then to continue nine years.

$$R_{50} \times (a_{\overline{50}|9} + 1) + a_{50}$$

To find the present value of the reversion of a perpetual annuity, after the failure—

1°. Of a single life (age 40; p, perpetuity) $(p - a_{40})$.

2°. Of two joint lives . . . $(p - a_{25.20})$.

3°. Of last of two lives . . . . . $(p - a_{\overline{70.75}}^{v})$.

Let $\angle_9$ represent present value of £1 per annum for nine years; then,

To find the present value of the reversion of what may remain of an annuity for a number of years certain, after the failure—

1°. Of an existing life (say 40 years of age) $(\angle_9 - a_{40})$.

2°. Of two joint lives (say 40 and 45) . . $(\angle_9 - a_{40.45})$.

3°. Of the last of two lives . . . . $(\angle_9 - a_{\overline{40.45}}^{v})$.

To find the present value of an annuity certain, for a given number of years, and afterwards to continue—

1°. During an existing life, $\angle_{10} + a_{\overline{\underset{10-1}{45}}|}$

2°. During two joint lives, $\left(\angle_{10} + a_{\overline{\underset{10-1}{35.45}}|}\right)$

3° During the last of two lives, $\left(\angle_{10} + a_{\overline{\underset{10-1}{\overline{35.45}}}|}\right)$.

To find the present value of an annuity to be divided equally between two persons during their joint lives, and to be reduced to one-half of the amount during the life of the survivor of them.

Let the ages be thirty-five and forty, then the formula becomes:

$$\frac{a_{35} + a_{40}}{2}$$

To find what sum each of two persons ought respectively to pay for an annuity to be equally divided between them, while they both continue to live, but which, on the death of either of them, is to belong entirely to the survivor.

Let the ages be respectively twenty-five and forty

$$\left(a_{25} - \frac{a_{25.40}}{2}\right) = \text{youngest person's contribution.}$$

$$\left(a_{40} - \frac{a_{25.40}}{2}\right) = \text{eldest person's contribution.}$$

To find the present value of that part of an annuity granted on the longest of two lives, which may remain after the death of either of them, to a third person and his heirs, during the life of the survivor.

Let the respective ages be twenty-five and thirty:

$$a_{25} + a_{30} - 2\,(a_{25.30}) = \left\{ \begin{array}{l} \text{present value of remainder of an-} \\ \text{nuity on death of either life.} \end{array} \right.$$

TABLE XI.

Decimals corresponding with every Pound in the Hundredweight.

Lbs.	Decimals.	Lbs.	Decimals.	Lbs.	Decimals.	Lbs.	Decimals.
1	·0089286	29	·2589286	57	·5089287	85	·7589285
2	·01785714	30	·2678571	58	·517857	86	·7678571
3	·02678572	31	·2767857	59	·5267857	87	·7767859
4	·03571429	32	·2857144	60	·5357143	88	·7857143
5	·04464286	33	·2946428	61	·5446428	89	·7946429
6	·05357143	34	·3035714	62	·5535715	90	·8035733
7	·0625	35	·3125	63	·5625	91	·8125
8	·07142856	36	·3214286	64	·5714286	92	·8214285
9	·08035715	37	·3303572	65	·5803571	93	·8303572
10	·08928571	38	·3392857	66	·5892856	94	·8392857
11	·09821430	39	·3482143	67	·5982142	95	·8482124
12	·1070689	40	·3571429	68	·6071429	96	·8571427
13	·1160714	41	·3660715	69	·6160714	97	·8660714
14	·125	42	·375	70	·625	98	·875
15	·1339287	43	·3839286	71	·6339285	99	·8839286
16	·1428542	44	·3928751	72	·6428571	100	·8928572
17	·1517857	45	·4017857	73	·6517857	101	·9017858
18	·1607141	46	·4107142	74	·6607143	102	·9107144
19	·1696420	47	·4196428	75	·669643	103	·9196428
20	·1785714	48	·4285713	76	·6785714	104	·9285714
21	·1875	49	·4375	77	·6875	105	·9375
22	·1964285	50	·4464286	78	·6964286	106	·9464286
23	·2053571	51	·4553572	79	·7053571	107	·9553572
24	·214285	52	·4642856	80	·7142873	108	·9642856
25	·2232144	53	·4732143	81	·7232143	109	·9732142
26	·2321428	54	·4821428	82	·732143	110	·982143
27	·2410714	55	·4910714	83	·7410715	111	·9910716
28	·25	56	·5	84	·75	112	1·0000000

VARIOUS CALCULATIONS.

70°. All mathematical problems are questions of quantity, and may be solved by comparing, in various ways, known quantities with unknown.

We have taken the symbol x as the representative of the amount of time, space, weight, &c., concerning which the question is asked; this symbol we place on the left side of the equation, and under it all the known or supposed terms, by which the opposite and similar terms on the right are to be measured or compared.

When the general question is—" If so much, how much

more?"—it is said to be in *direct* proportion, and there is no irregularity in the statement; but if, on the other hand, the question be—"If so much, how much less?"—the proportion is *indirect*, and quantities placed on the right, to be measured by the left, must be transposed.

It is difficult to frame a rule sufficiently abstract to meet *all* those cases—numerical enigmas—with which school arithmetics are usually filled; of rare occurrence, indeed, in actual business, but deemed valuable as exercises, or illustrations of a principle. For the last-mentioned purpose, we select a few such examples, containing a large number of terms. The following is the method most frequently used in forming statements of this kind.

Write all the known or supposed terms in one line, and under them the respective similar terms, concerning which the question is asked. The place of x will instantly be found under the corresponding term; then arrange the two columns by placing x on the left, opposite the known term of the same sort, as,

x days 20 days;

then in the left column write all the *known* terms, and opposite to them, on the right, all the similar terms which are to be measured by them. Compare now all the terms with x, and if in any case the answer be in opposition to the question, as, "the more the one, the less the other," the numbers must be transposed; for example:

If 15 men can dig a field 36 yards broad and 108 yards long in 9 days, working 8 hours a day, in how many days can 24 men, working 10 hours per day, dig a field 84 yards broad and 288 yards long?

Men.	broad.	long.	days.	hours.	
15	36	108	9	8	known terms.
24	84	288	x	10	unknown, and connected with x.

x days	9 days.	
8 hours	10 hours.	The more hours, the fewer days: transpose.
108 long	288 long.	
36 broad	84 broad.	
15 men	24 men.	The more men, the fewer days · transpose

True Statement.

$$\left.\begin{matrix} x \\ 10 \\ 108 \\ 36 \\ 24 \end{matrix}\right\} = \left\{\begin{matrix} 9 \text{ days} \\ 8 \text{ hours} \\ 288 \text{ long} \\ 84 \text{ broad} \\ 15 \text{ men} \end{matrix}\right\} 28 \text{ days, answer.}$$

Or the question may be stated as two equations, the first formed of the *known* terms, and the second of the *unknown* term and those connected with it, thus:—

$$\left.\begin{matrix} 36 \text{ broad} \\ 108 \text{ long} \end{matrix}\right\} = \left\{\begin{matrix} 15 \text{ men} \\ 9 \text{ days} \\ 8 \text{ hours} \end{matrix}\right\} \text{known terms.}$$

$$\left.\begin{matrix} x \text{ days} \\ 10 \text{ hours} \\ 24 \text{ men} \end{matrix}\right\} = \left\{\begin{matrix} 84 \text{ broad} \\ 288 \text{ long} \end{matrix}\right\} \text{unknown, \&c.;}$$

which, worked together as the former statement, will evidently produce the same result.

Again, without considering whether the proportions be direct or inverse, the terms may simply be set down according to the logical sequence of the ideas, as in the example below, which should be read down the left column and then down the right.

x—In how many days of 10 hours each can 24 men dig 36 yards broad and 108 long,	if in 9 days of 8 hours each 15 men can dig 84 yards broad and 288 long.

Examples.

71°. If 84 men reap 72 acres in 15 days, how many acres will 96 men reap in 12 days?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 96 \text{ men} \\ 12 \text{ days.} \end{matrix}\right.$$

$$\left.\begin{matrix} 84 \text{ men} \\ 15 \text{ days} \end{matrix}\right\} = 72 \text{ acres.}$$

Measure both sides by 12; ∴ $\left.\begin{matrix} x \\ 7 \\ 15 \end{matrix}\right\} = \left\{\begin{matrix} 8 \\ 12 \\ 72 \end{matrix}\right\} = \overset{\text{ac.}}{65}\ \overset{\text{ro.}}{3}\ \overset{\text{po.}}{12\tfrac{4}{7}}$

If 10 months' provision for 180 boys cost £3760, what will be the cost of 12 months' provision for 96 boys?

$$\left.\begin{array}{r} x \\ 180 \\ 10 \end{array}\right\} = \left\{\begin{array}{r} 3760 \\ 96 \\ 12 \end{array}\right. \quad \text{Measure both sides by 10 and 12.} \quad \left.\begin{array}{r} x \\ 15 \\ 1 \end{array}\right\} = \left\{\begin{array}{r} 376 \\ 8 \\ 12 \end{array}\right\} = £2406\ 8s.$$

What will be the value of $\frac{9}{14}$ yard, if $\frac{3}{4}$ yard cost $£\frac{7}{18}$?

$$\left.\begin{array}{r} x \\ \frac{3}{4}\text{ yd.} \\ £1 \end{array}\right\} = \left\{\begin{array}{l} \frac{9}{14}\text{ yd.} \\ £\frac{7}{18} \\ 20s. \end{array}\right.$$

$$\text{Transpose the denominators.} \left.\begin{array}{r} 14 \\ 18 \\ 3 \end{array}\right\} = \left\{\begin{array}{l} 4 \\ 9 \\ 7 \\ 20 \end{array}\right. \quad \text{Divide by 3, the common measure.} \quad \left.\begin{array}{r} 14 \\ 6 \end{array}\right\} = \left\{\begin{array}{r} 4 \\ 7 \\ 20 \end{array}\right\} = 6s.\ 8d.$$

A can reap a field in 12 days, B in 6, and C, in 4 days; in what time can they all do it together?

To find the time required for A, B, and C unitedly to reap the field, the work must be measured by what they can all do together in one day.

Now A reaps $\frac{1}{12}$, B $\frac{1}{6}$, and C $\frac{1}{4}$, in one day. These fractions collected by reduction and addition, amount to $\frac{12}{24}$; ∴

What time will the work require if A, B, and C can perform $\frac{12}{24}$ of it in one day?

$$\left.\begin{array}{r} x \\ \frac{1}{12}+\frac{1}{6}+\frac{1}{4}=\frac{12}{24} \end{array}\right\} = \left\{\begin{array}{l} 1\text{ work} \\ 1\text{ day} \end{array}\right.$$

$$\text{Transpose denominator.} \quad \left.\begin{array}{r} x \\ 12 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ 24 \end{array}\right\} = \tfrac{24}{12},\text{ or 2 days.}$$

One pipe alone will fill a cistern in 9 hours, a second alone in 6 hours; in what time can the cistern be filled by the two running together?

$$\left.\begin{array}{r} x \\ \frac{1}{9}+\frac{1}{6}=\frac{15}{54} \end{array}\right\} = \left\{\begin{array}{lcr} 1 & & x \\ & = & \\ 1\text{ hour} & & 15 \end{array}\right\} = \left\{\begin{array}{l} 1 \\ 54 \end{array}\right\} \tfrac{54}{15},\text{ or } 3\tfrac{3}{5}\text{ hours.}$$

If 15 persons can be maintained for 60 days on £80, how much money will support 96 persons for 300 days?

$$\left.\begin{array}{r} x \\ 15 \\ 60 \end{array}\right\} = \left\{\begin{array}{r} 80 \\ 96 \\ 300 \end{array}\right\} = £2560$$

What is the commission on £840 9*s*., at £$\frac{3}{4}$ per cent?

$$\left.\begin{matrix} x \\ 100 \end{matrix}\right\} = \left\{\begin{matrix} 840\cdot45 \\ \frac{3}{4} \end{matrix}\right\} = £6\ 6s.\ 0\tfrac{3}{4}d.$$

What is the expense of insuring a cargo valued at £754; premium three guineas per cent., policy duty 5*s*. per cent.?

Reckon policy duty on £800 = £2.

$$\left.\begin{matrix} \text{Commission on £400} = 10s. \\ \text{Premium} \quad . \quad £3\ 3s. \end{matrix}\right\} = 3\cdot65.$$

$$\therefore \left.\begin{matrix} x \\ 100 \end{matrix}\right\} = \left\{\begin{matrix} 754 \\ 3\ 65 \end{matrix}\right\} = 27\cdot521 + 2 \text{ duty, } i.\ e., £29\ 10s.\ 5\tfrac{1}{4}d.$$

The average rate at which 3 per cent. stock was created, by the various loans contracted from 1792 to 1802, was, in whole numbers, £57; what is the cost of a loan of £3,000,000 at this rate?

$$\left.\begin{matrix} x \\ 57 \end{matrix}\right\} = \left\{\begin{matrix} 3{,}000{,}000 \\ 100 \end{matrix}\right\} = £5{,}263{,}157\tfrac{17}{19}, \text{ and 3 per cent. interest.}$$

Multiply $\frac{2}{3}$ by $\frac{3}{4}$. $\quad x \left| \begin{matrix} \frac{2}{3} \\ \frac{3}{4} \end{matrix} \right.$ transpose $\left.\begin{matrix} 3 \\ 4 \end{matrix}\right\} = \left\{\begin{matrix} 2 \\ 3 \end{matrix}\right. = \frac{6}{12}$, or $\frac{1}{2}$.

Divide $\frac{4}{5}$ by 7. $\quad \begin{matrix} x \\ 7 \end{matrix} \left| \ \frac{4}{5} = \left.\begin{matrix} 7 \\ 5 \end{matrix}\right\} = \left\{\begin{matrix} 4 \\ 0 \end{matrix}\right. = \frac{4}{35}. \right.$

Required the amount of $12\frac{2}{5}$ multiplied by $\frac{7}{8}$.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 12\frac{2}{5} \text{ or } \frac{62}{5}, \\ \frac{7}{8} \end{matrix}\right. \therefore \left.\begin{matrix} x \\ 5 \\ 8 \end{matrix}\right\} = \left\{\begin{matrix} 62 \\ 7 \div 2 = \left\{\begin{matrix} 5 \\ 4 \end{matrix}\right\} \end{matrix}\right. = \left\{\begin{matrix} 31 \\ 7 \end{matrix}\right. = 10\tfrac{17}{20}.$$

How many pieces of eight at 56*d*. each, will answer a bill of £594 6*s*. sterling?

$$\left.\begin{matrix} x \\ 233\dot{3} \end{matrix}\right\} = \left\{\begin{matrix} 594\cdot3 \\ 1 \end{matrix}\right\} = 2547$$

Reduce 6 furlongs 16 poles to the fraction of a mile

$$\left.\begin{matrix} x \\ 1 \text{ fur.} \end{matrix}\right\} = \left\{\begin{matrix} 6 \text{ fur. } 16 \text{ po.} \\ 40 \text{ po.} \\ 1 \text{ mile} \end{matrix}\right\} = \tfrac{256}{320}, \text{ or } \tfrac{4}{5}$$

If 12 yards at London make 8 ells at Paris, how many ells at Paris will make 64 yards at London?

$$\left.\begin{matrix} x \\ 12 \text{ yds.} \end{matrix}\right\} = \left\{\begin{matrix} 64 \text{ yds.} \\ 8 \text{ ells} \end{matrix}\right\} = 42\tfrac{8}{12} \text{ ells}$$

Reduce $\frac{5}{7}$ to a fraction of the same value, whose denominator shall be 35.

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 35 \\ \frac{5}{7} \end{matrix}\right. \quad \text{Transpose} \left.\begin{matrix} x \\ 7 \end{matrix}\right\} = \left\{\begin{matrix} 35 \\ 5 \end{matrix}\right\} = \tfrac{25}{35}.$$

A thousand quills cost eight shillings, at what price must they be sold to clear 20 per cent. and give six months' credit? *i. e.*,

What profit on 8*s*., if 100 gain £22½?

$$\left.\begin{matrix} x \\ 100 \end{matrix}\right\} = \left\{\begin{matrix} \cdot 4 \\ 22 \cdot 5 \end{matrix}\right\} = \cdot 09 \text{ profit} + \cdot 4 = 9s.\ 9\tfrac{3}{4}d.$$

If 2 cwt. 1 qr. 7 lb. cost £8 14*s*. 4*d*., what will 14 cwt. 3 qrs. cost?

$$\left.\begin{matrix} x \\ 2 \cdot 3126 \end{matrix}\right\} = \left\{\begin{matrix} 8 \cdot 716\dot{6} = \log 0 \cdot 9403501 \\ 14 \cdot 75 \quad = \log 1 \cdot 1687920 \end{matrix}\right.$$

2·1091421
0·3640817 deduct log of divisor.

1·7450604 = 55·598, or
£55 11*s*. 11¾*d*.

By the usual method, £55 11*s*. 11½ $\frac{10}{37}$*d*.

What is the value of £$\frac{5}{13}$?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} \frac{5}{13} \\ 20s. \end{matrix}\right. \quad \therefore \left.\begin{matrix} 13 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 5 \\ 20 \end{matrix}\right\} 7s.\ 8\tfrac{1}{4}\ \tfrac{3}{13}d.$$

Reduce 3*s*. 3*d*. to the fraction and the decimal of a pound

$$\left.\begin{matrix} x \\ 1s. \\ 240d. \end{matrix}\right\} = \left\{\begin{matrix} 3s\ \ 3d. \\ 12d. \\ £1 \end{matrix}\right\} £\tfrac{13}{80}. \quad \left.\begin{matrix} x \\ 240d. \end{matrix}\right\} = \left\{\begin{matrix} 39d. \\ £1 \end{matrix}\right\} = \cdot 1625$$

Reduce $\frac{2}{630}$ of £1 to the fraction of a penny

$$\left.\begin{matrix} x \\ £1 \end{matrix}\right\} = \left\{\begin{matrix} \frac{2}{630} \\ 240d. \end{matrix}\right. \quad \therefore \left.\begin{matrix} 630 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 2 \\ 240 \end{matrix}\right\} \tfrac{48}{63}d$$

The sum of £115 was paid for income tax; what was the amount of the income, rate of interest 3 per cent.?

$$\left.\begin{matrix} x \\ 3 \end{matrix}\right\} = \left\{\begin{matrix} 115 \\ 100 \end{matrix}\right\} = £3833\ 6s.\ 8d.$$

If the cost of a cwt. be 47*s.*, how much is that per lb.?

$$\left.\begin{matrix} x \\ 112\text{ lbs.} \\ 1s. \end{matrix}\right\} = \left\{\begin{matrix} 1\text{ cwt.} \\ 47s. \\ 48\text{ farthings.} \end{matrix}\right.$$

Measure each side by the common divisor 16

$$\left.\begin{matrix} x \\ 7 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ 47 \\ 3 \end{matrix}\right\} = 20\tfrac{1}{7}\text{ farthings per lb.}$$

which illustrates the rule—"To find the price per lb., multiply the price of the cwt. in shillings by 3, divide the product by 7; the quotient will be the price of 1 lb. in farthings."

Remitted from London to Amsterdam, a bill of £754 10*s* sterling; how many pounds Flemish is the sum; the exchange at 33*s*. 6*d*. Flemish per pound sterling?

$$\left.\begin{matrix} x \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 754{\cdot}5 \\ 1{\cdot}675 \end{matrix}\right\} = £1263\ 15s.\ 9d.$$

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$$\left.\begin{matrix} x \\ £1 \\ 1\text{ yr.} \end{matrix}\right\} = \left\{\begin{matrix} {\cdot}00416 \\ (1{\cdot}03)^1 \\ 1850\text{ yrs.} \end{matrix}\right\} =$$

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